Ground Truth

Low Distortion Projections for Surveying and GIS

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Where we are going…
…and how we will get there

- The necessary nuisance of **map projections**
  - An ***essential*** part of GPS surveying & mapping, and GIS
- Map projection distortion — what is it?
- Methods for reducing map projection distortion
  - The pitfalls of “modified” State Plane
  - Design of Low Distortion Projections (LDPs)
- LDP design example
- Spatial data documentation (*metadata!*)
- ***Ask questions at any time!***

Why do we need to worry about coordinate systems?

Because our society and economy are becoming increasingly ***"spatially enabled"***

iPoo - ***the GPS toilet locator***

Some questions to think about

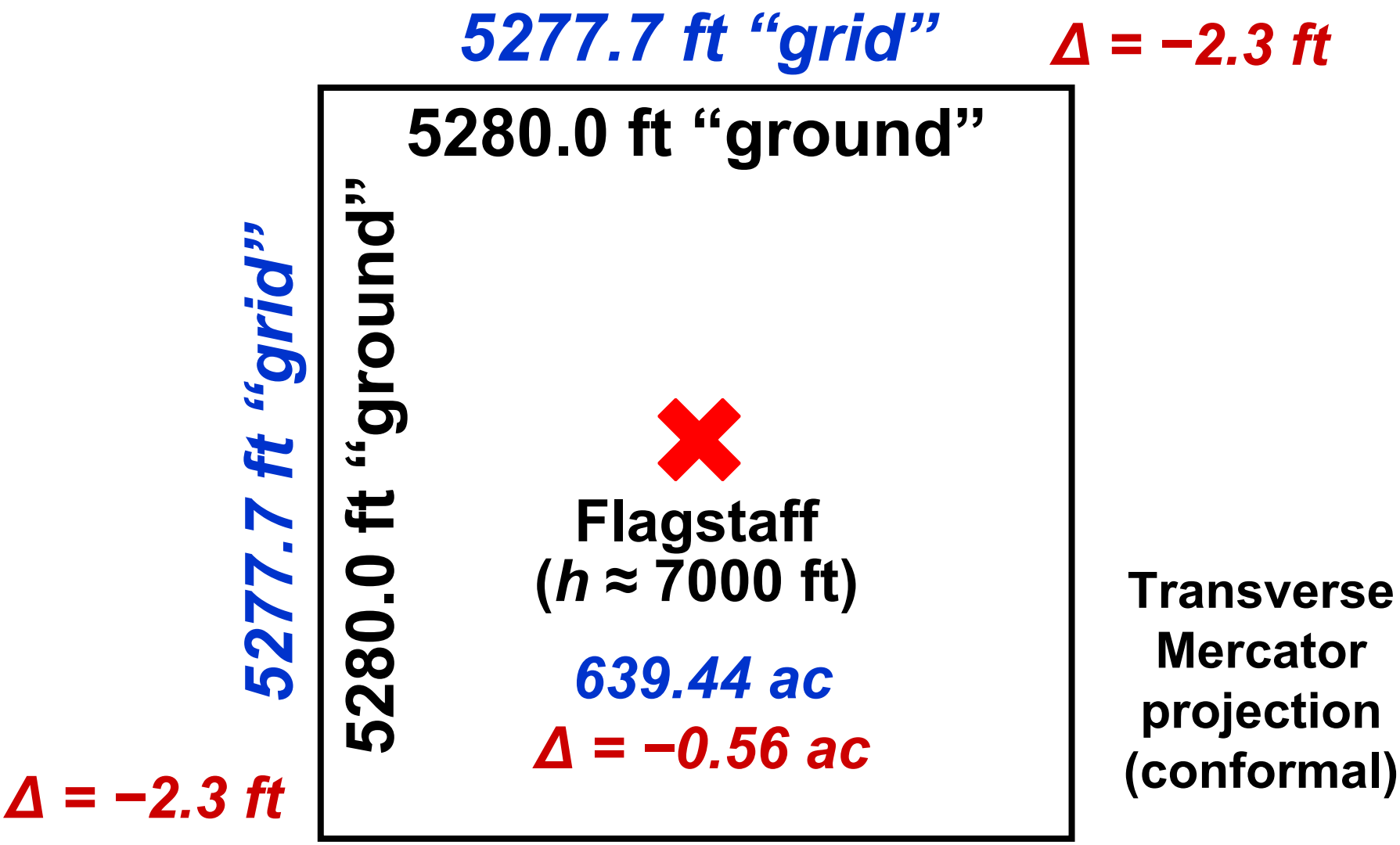
- What is a “coordinate system”?
  - ***Coordinate system***
    = <u>map projection</u> + geodetic datum
- What is the <u>*purpose*</u> of a map projection?
- Should you define a projection for your local area? If so, what kind would work best?
- Are you interested in <u>*direct*</u> importation of digital plats, plans, and as-builts into GIS?
- Can you have more than one coordinate system for an area in a GIS?
- What if you want a new coordinate system but have data in some other coordinate system?
- Why not just use State Plane? Or UTM? Or some other published projection?
  - And what about “modified” State Plane…?

Let's think about State Plane

- When was it created?
- Where was it first used?
- ***Why*** was it created?
- Why and how is it used today?
  - Advantages: Standardized (and ***conformal***)
  - Several states have or are moving to single-zone system
- Disadvantages
  - Distortion often too great for "ground" distances
  - Often "modified" to get "ground" coordinates
  - No standardized way to "modify"
  - "Modification" will generally ***NOT*** minimize distortion

Map Projection Distortion

AZ State Plane of 1983, Central Zone (0202)



Map Projection Distortion

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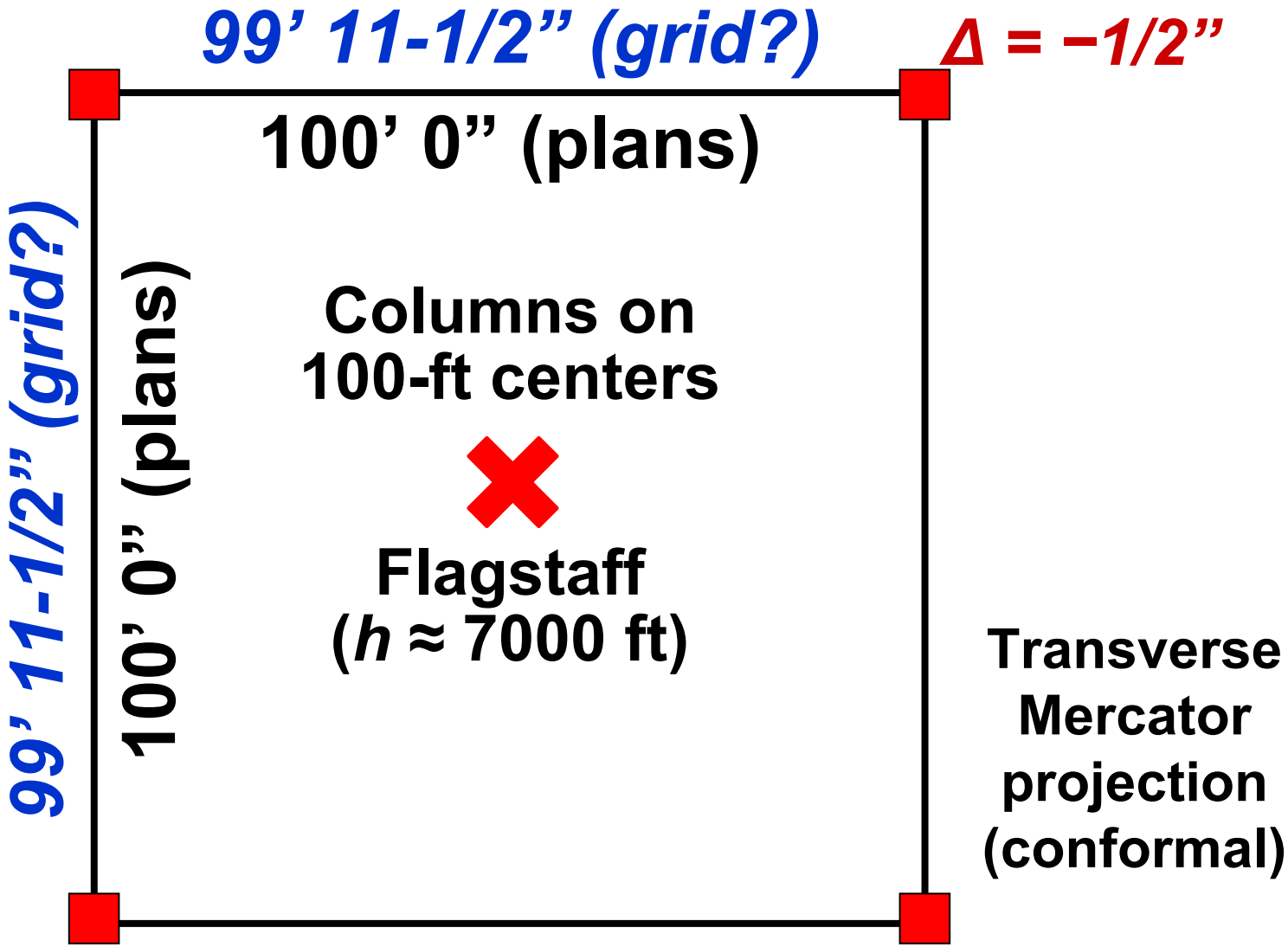
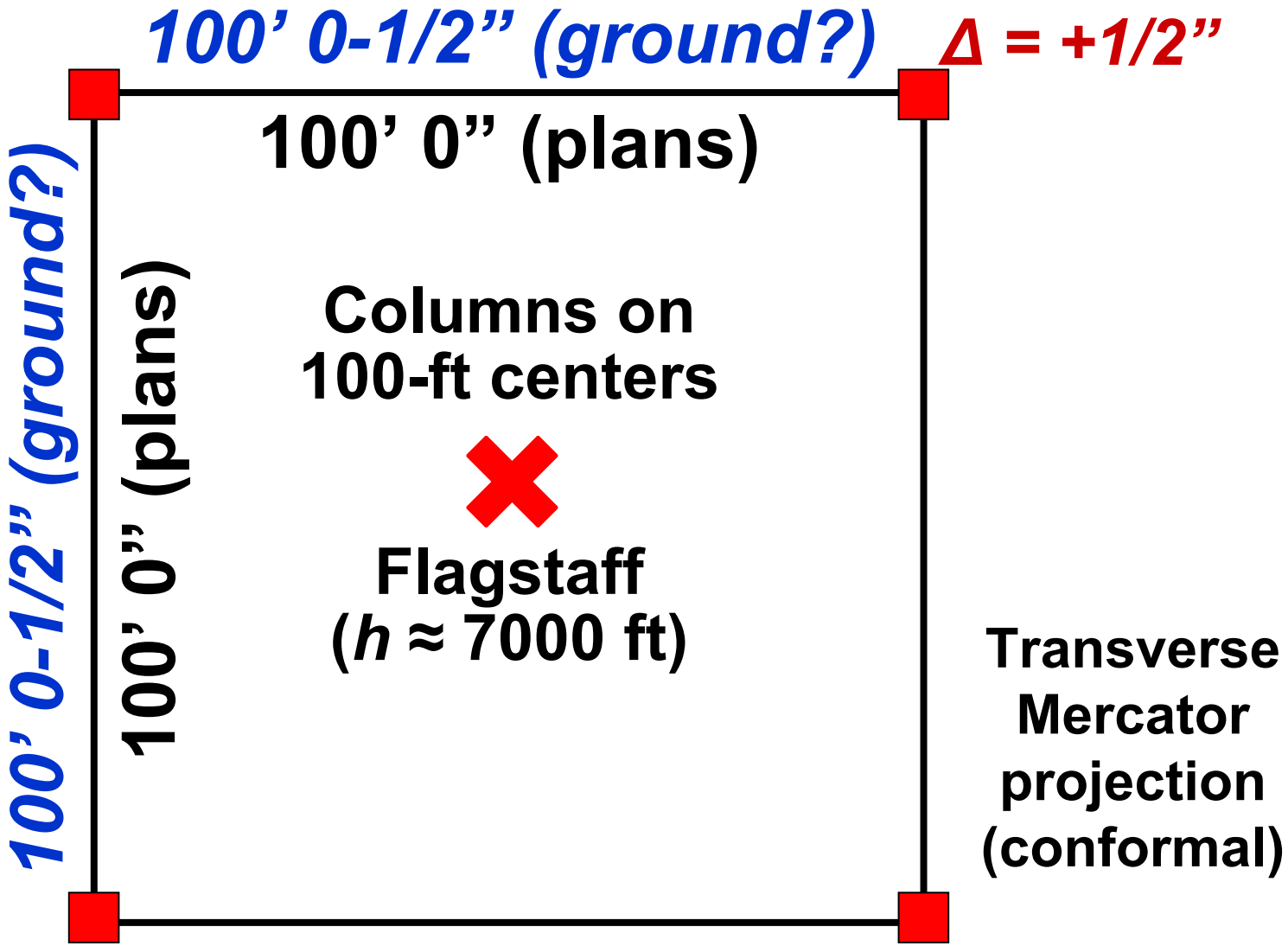


Map Projection Distortion

AZ State Plane of 1983, Central Zone (0202)



Why bother with LDPs?

- Provides standardized method for minimizing distortion over largest area possible
- Satisfies needs of both the surveying and GIS communities
- Enables direct use of survey data in a GIS
  - ...without resorting to “rubbersheeting” acts of desperation
- Reduces proliferation of local systems
  - Provides alternative for those who want (or need) “ground” coordinates
- Facilitates data transferability
  - Local data would be readily available to others (e.g., emergency services, Homeland Security)

The Problem

- Many users want a *"low-distortion"* map product
  - That is, they want distances on the map to equal the true horizontal "ground" distances
  - Strictly speaking, this is impossible
    - No such thing as "ground" coordinates
    - But distortion can be ***minimized***
- "Low distortion" is especially desirable for certain types of products, such as:
  - Engineering design and construction plans, as-builts
  - Boundary survey plats and legal descriptions
- This problem has *always* existed
  - But GPS has made it ***much*** more apparent . . .

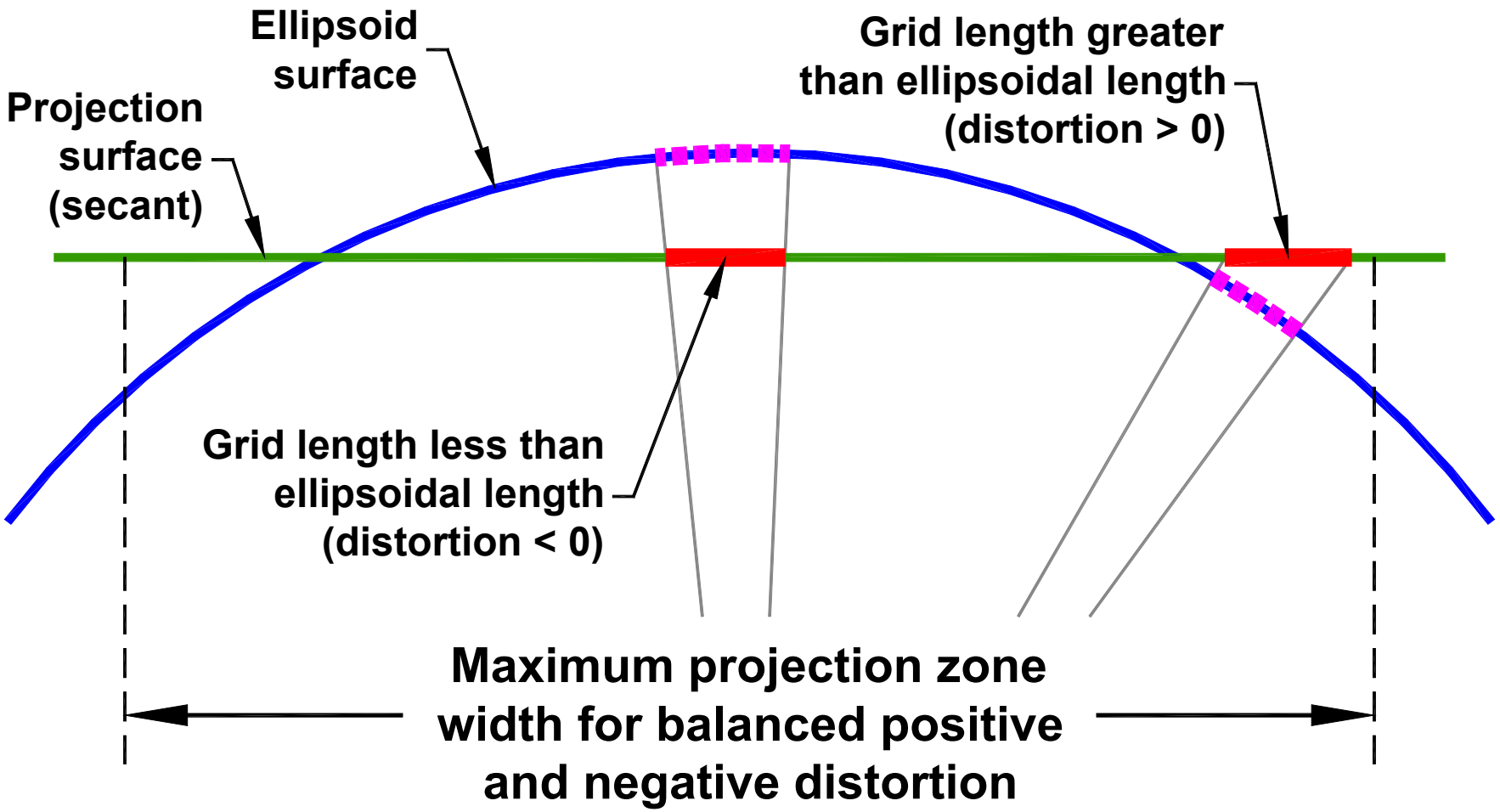
GPS & Map Projections

- GPS yields ***geodetic coordinates***
  - Usually these are expressed as *latitude*, *longitude*, and *ellipsoid height*
- These ***MUST*** be transformed to be useable
  - Converted to "grid" coordinates using a "*map projection*"
    - i.e., *Northing*, *Easting*, "*Elevation*" (*y*, *x*, "*z*")
  - *Essential* for printed maps, computer display, CAD
  - Projected coordinates are ***always*** distorted
    - There is no way around this — it is a ***Fact of Life***
    - Cannot be eliminated — it can only be reduced
    - Effect is computable — it is **NOT** the same as "error"

Types of Map Projection Distortion

- **Angular distortion.** Equals *convergence (mapping) angle* for *conformal* projections (e.g., Transverse Mercator)
  - Effect increases with increasing latitude
    - ~35” per mile east-west in central Arizona
  - Usually not as much of a concern as . . .
- **Linear distortion.** Difference in distance between a pair of grid (map) coordinates when compared to the “true” horizontal (“ground”) distance
  - Can express as ratio of distorted length to “true” length
    - E.g., as feet of distortion per mile, or as mm/km (= ppm)
  - Distortion can be **negative** or **positive**
    - **NEGATIVE:** Grid distance less than true distance
    - **POSITIVE:** Grid distance greater than true distance
  - *<u>NOTE</u>: Minimizing linear distortion only makes sense for **conformal** projections*

Linear distortion due to Earth curvature



Horizontal distortion due to Earth curvature

Maximum projection zone width	**Maximum linear horizontal distortion**		
	Parts per million	Feet per mile	Ratio (absolute value)
16 miles	±1 ppm	±0.005 ft/mi	1 : 1,000,000
50 miles	±10 ppm	±0.05 ft/mi	1 : 100,000
71 miles	**±20 ppm**	**±0.1 ft/mi**	**1 : 50,000**

Horizontal distortion due to Earth curvature

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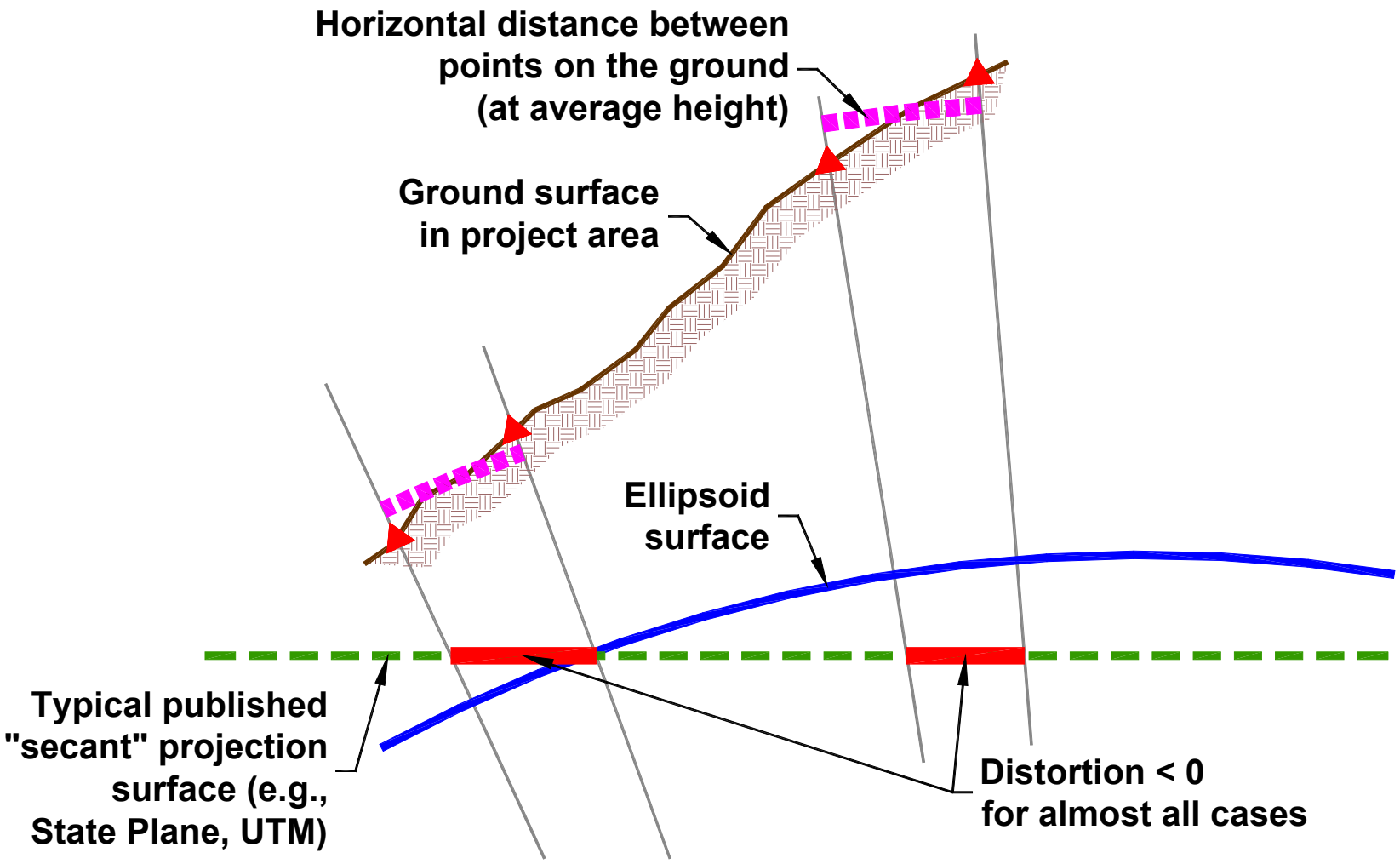
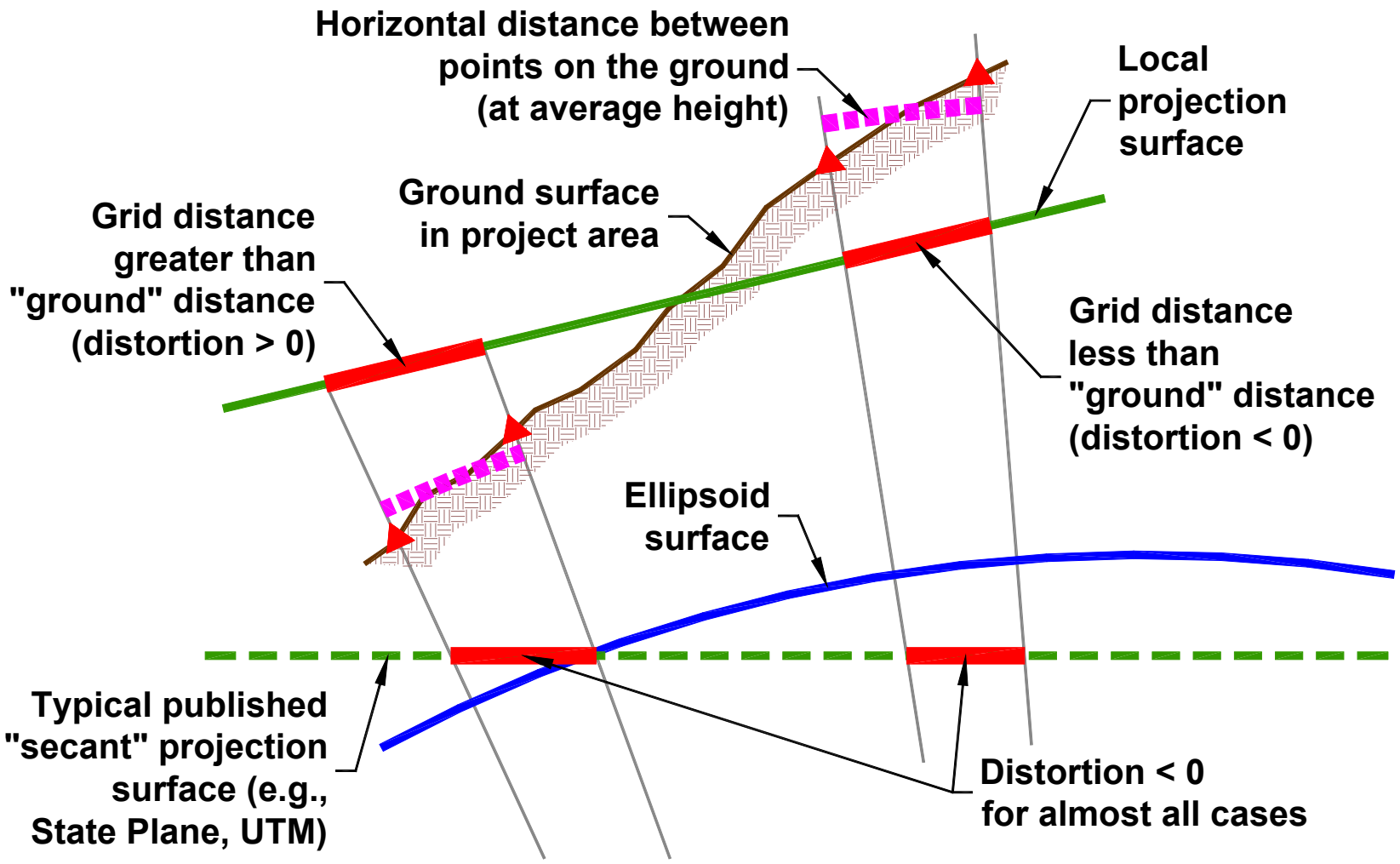
	Parts per million	Feet per mile	Ratio (absolute value)
16 miles	±1 ppm	±0.005 ft/mi	1 : 1,000,000
50 miles	±10 ppm	±0.05 ft/mi	1 : 100,000
71 miles	**±20 ppm**	**±0.1 ft/mi**	**1 : 50,000**
112 miles	±50 ppm	±0.3 ft/mi	1 : 20,000
158 miles*	±100 ppm	±0.5 ft/mi	1 : 10,000
317 miles**	±400 ppm	±2.1 ft/mi	1 : 2,500

Linear distortion due to ground height above ellipsoid



Linear distortion due to ground height above ellipsoid



Horizontal distortion due to height above ellipsoid

Height below (–) and above (+) projection surface	**Maximum linear horizontal distortion**		
	Parts per million	Feet per mile	Ratio (absolute value)
–100 ft, +100 ft	±4.8 ppm	±0.03 ft/mi	~1 : 209,000
–400 ft, +400 ft	**±19 ppm**	**±0.1 ft/mi**	**~1 : 52,000**
–1000 ft, +1000 ft	±48 ppm	±0.3 ft/mi	~1 : 21,000

Horizontal distortion due to height above ellipsoid

Height below (–) and above (+) projection surface	**Maximum linear horizontal distortion**		
	Parts per million	Feet per mile	Ratio (absolute value)
–100 ft, +100 ft	±4.8 ppm	±0.03 ft/mi	~1 : 209,000
–400 ft, +400 ft	**±19 ppm**	**±0.1 ft/mi**	**~1 : 52,000**
–1000 ft, +1000 ft	±48 ppm	±0.3 ft/mi	~1 : 21,000
+2000 ft	–96 ppm	–0.5 ft/mi	~1 : 10,500
+4000 ft*	–191 ppm	–1.0 ft/mi	~1 : 5,200
+7000 ft	–335 ppm	–1.8 ft/mi	~1 : 3,000

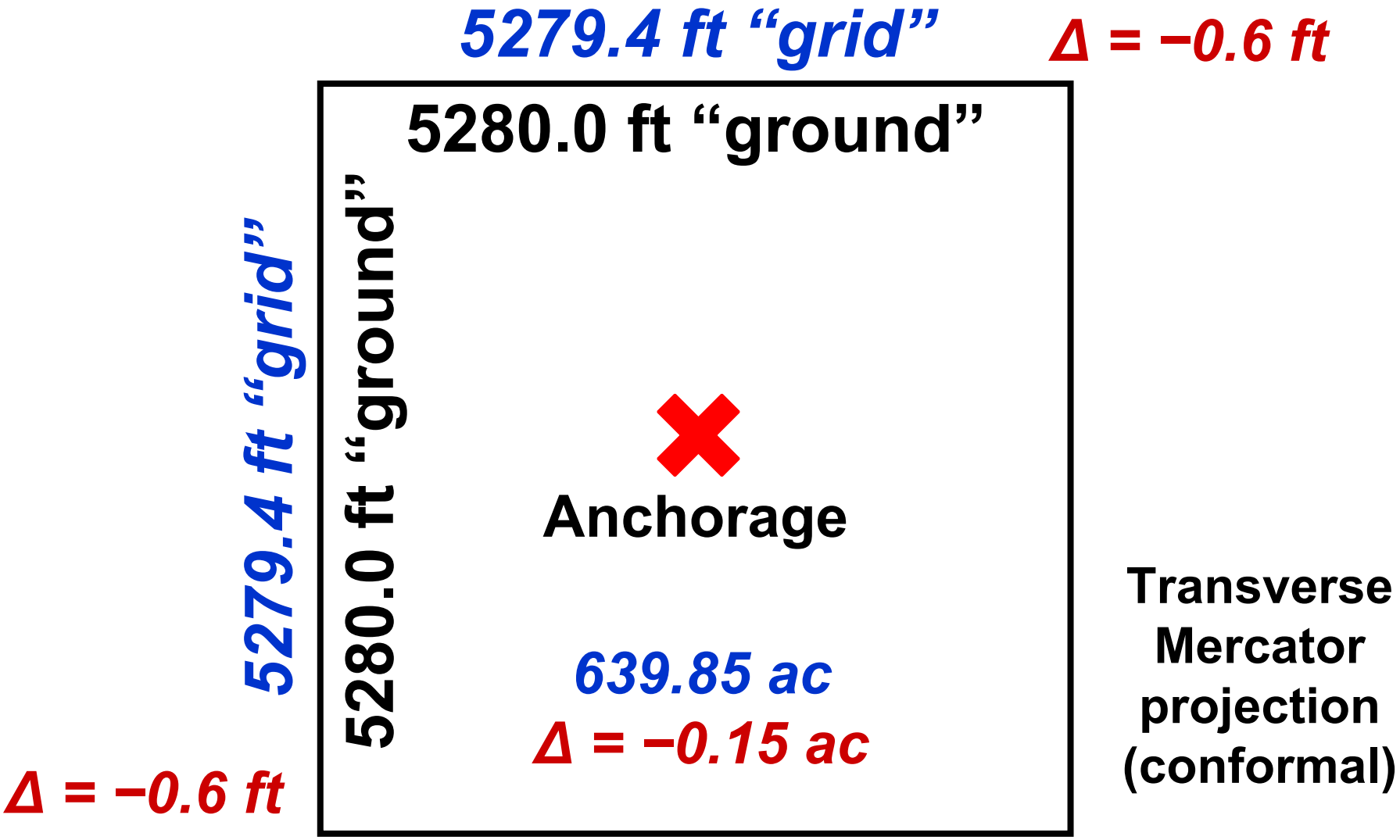
Conformality

What it is and why it matters

- ***Conformality:*** Scale (distortion) is the same in every direction at a point
  - i.e., the scale factor is unique at every point
  - Changes with direction for *ALL* other projections
- Angles on Earth are same as on map
  - Simple relationship between grid and geodetic azimuths
  - Tends to preserve shape
  - Important for surveying applications
- Keep in in mind that a projection is a *compromise*
  - Some characteristic must be “sacrificed”
  - e.g., no projection can be both conformal and equal area
- Conformal projections are the ***only*** type that make sense for “low distortion”

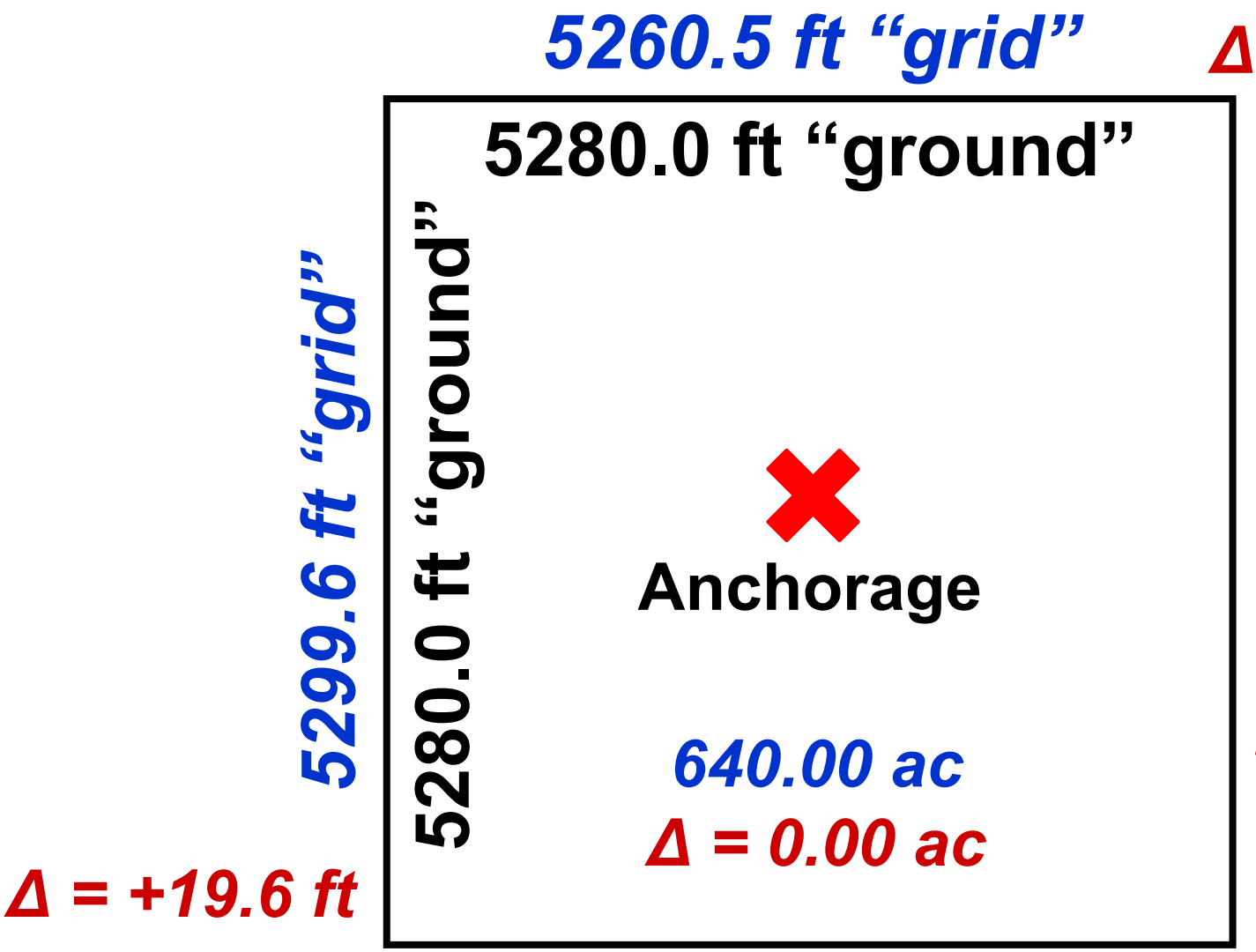
Map Projection Distortion

Alaska State Plane of 1983, Zone 4 (5004)



Map Projection Distortion

Alaska Albers Equal Area Projection



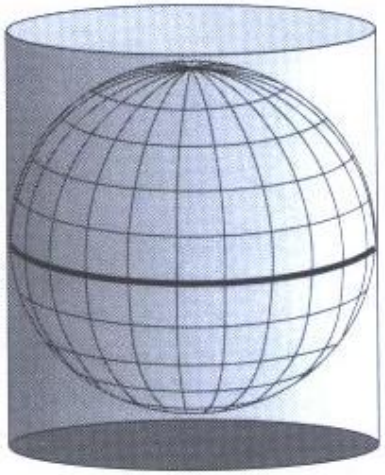
Distortion (scale) varies with direction for non-conformal projections

!

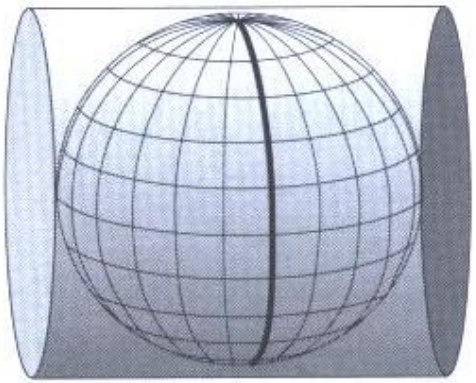
Common Types of Conformal Map Projections

- Best to use ones supported by a wide range of software
- *Cylindrical*
  - **<u>Transverse Mercator</u> (State Plane, UTM, USNG)**
  - "Regular" Mercator
  - Oblique Mercator (State Plane for AK panhandle)
- *Conical*
  - **<u>Lambert Conformal Conic</u> (State Plane)**
- *Azimuthal ("planar")*
  - Stereographic (all aspects)
  - Modified-Stereographic Conformal

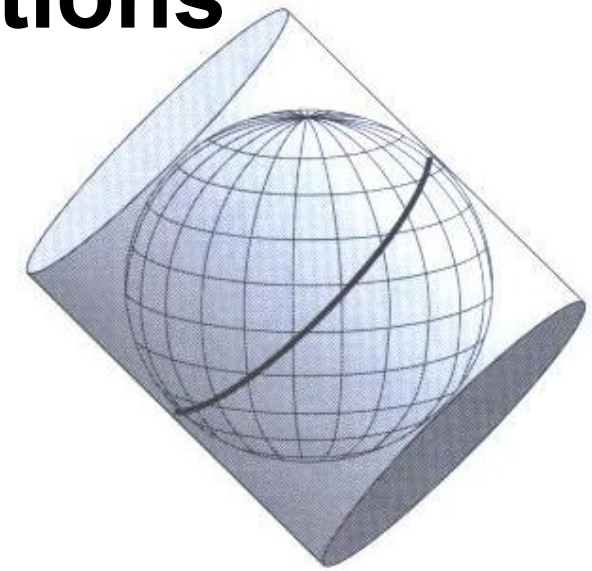
Cylindrical Projections



Normal



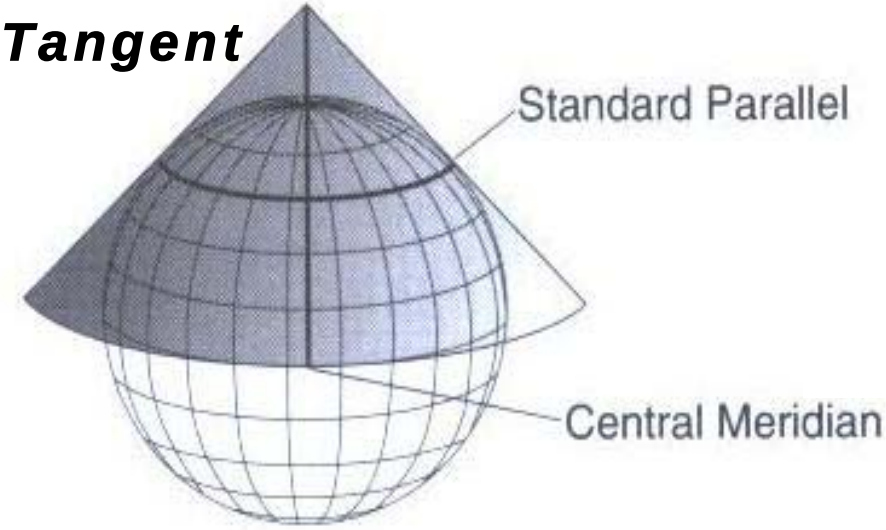
Transverse



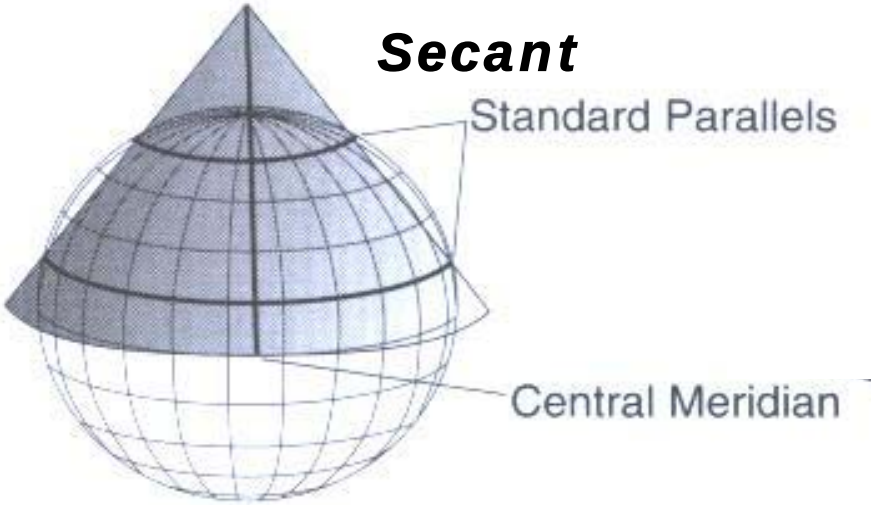
Oblique

Conical Projections

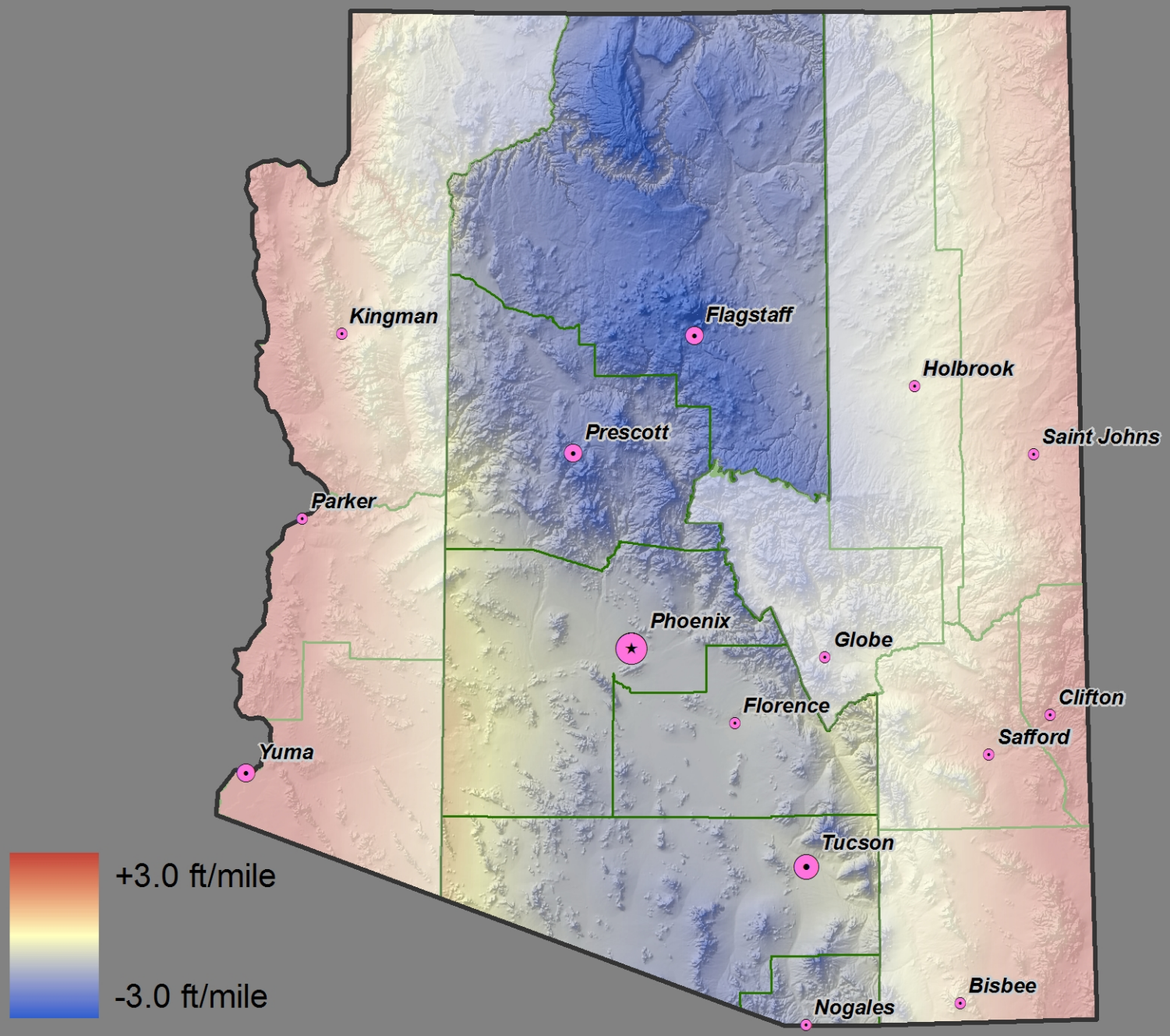
Tangent



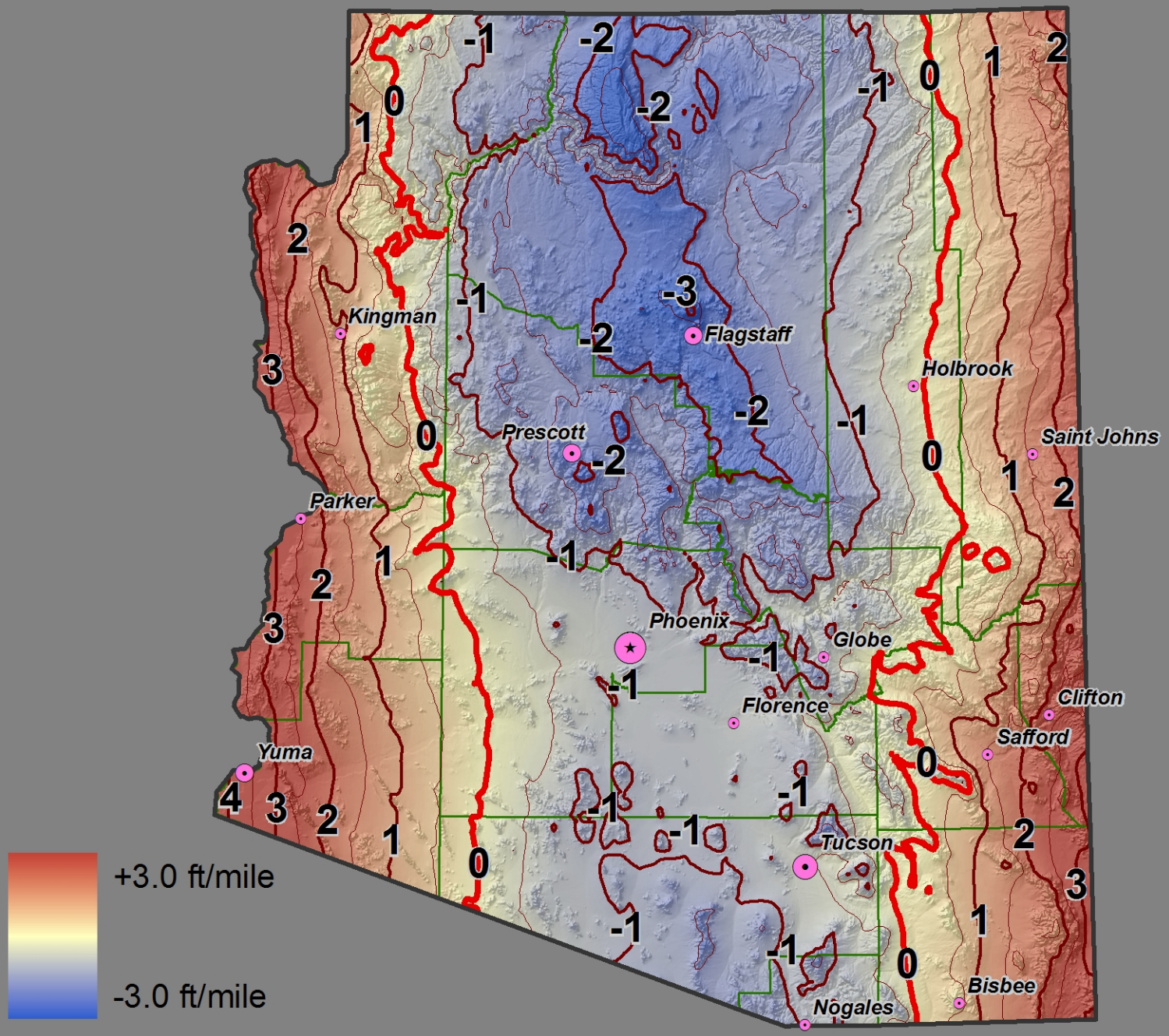
Secant



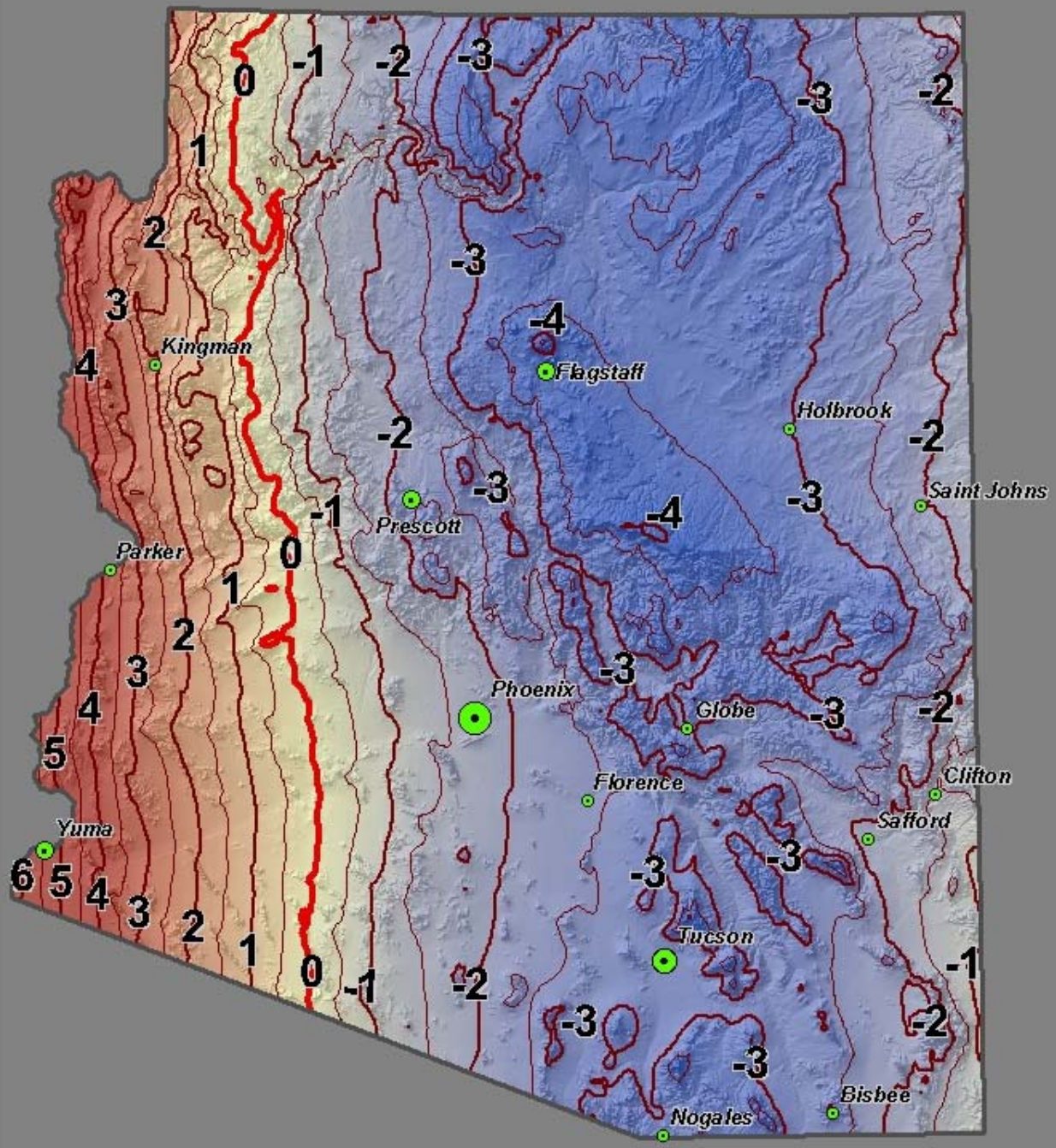
State Plane (AZ zone C) “linear distortion”



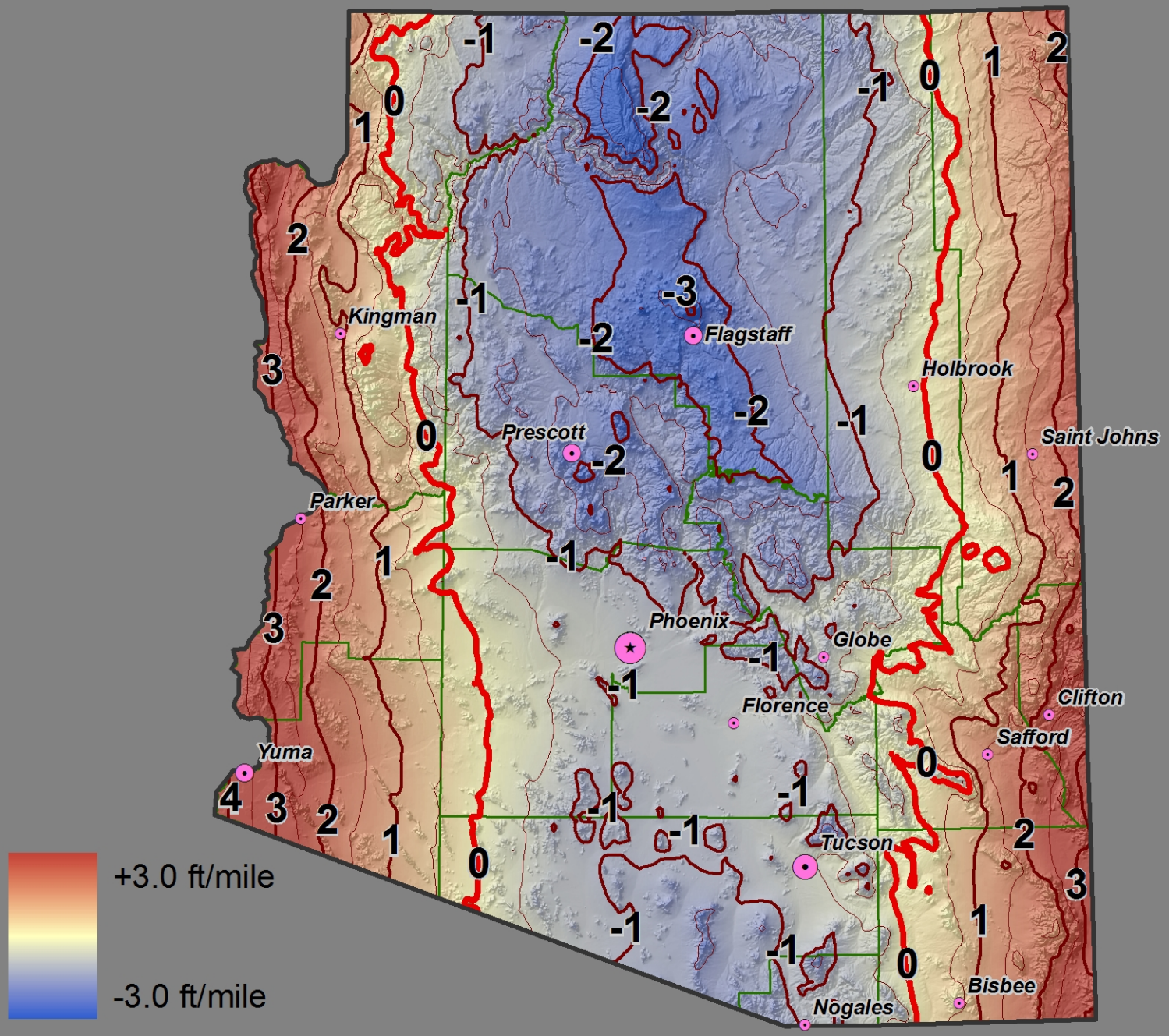
State Plane (AZ Zone C) linear distortion (feet/mile)



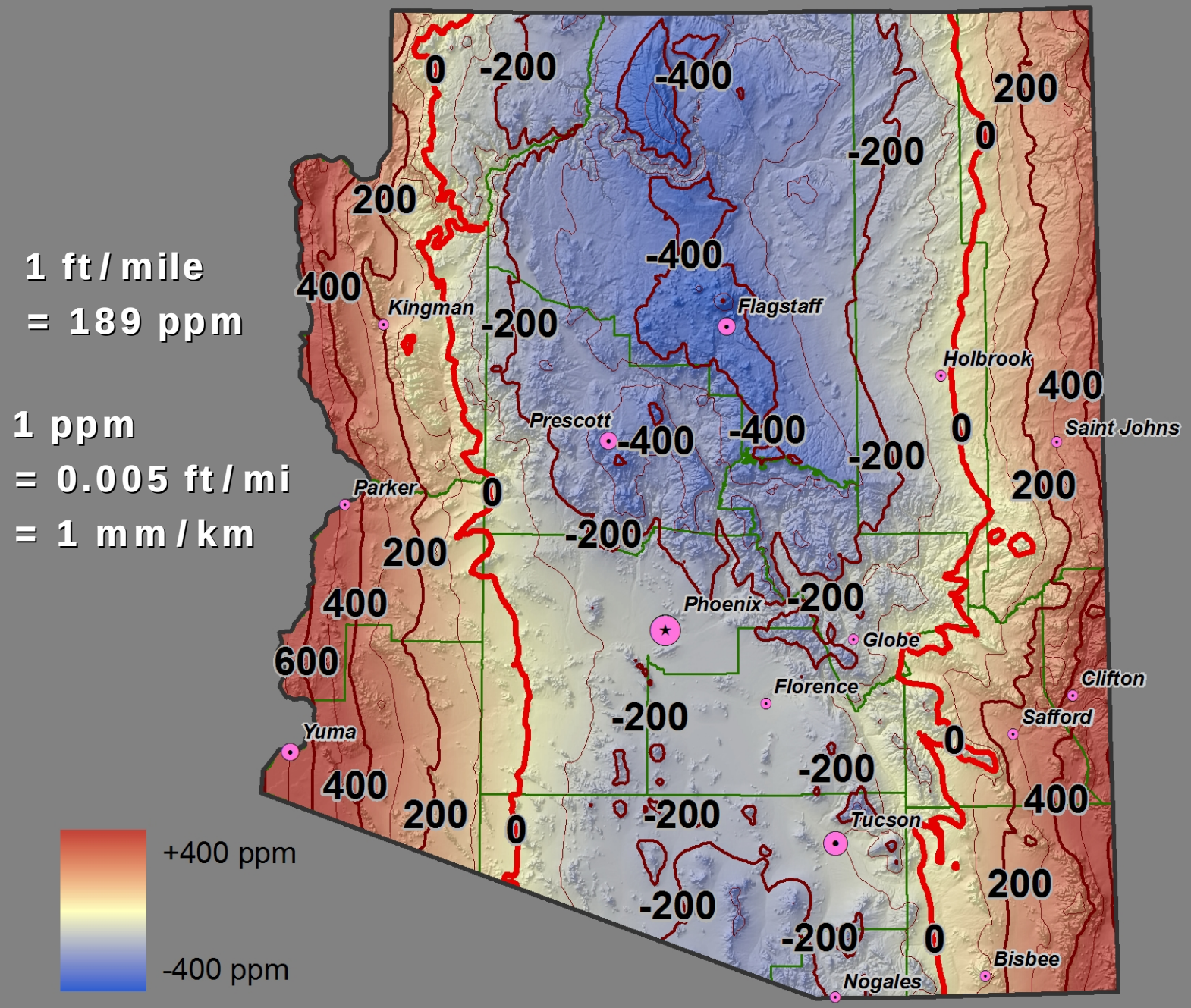
UTM (Zone 12 North) linear distortion (feet/mile)



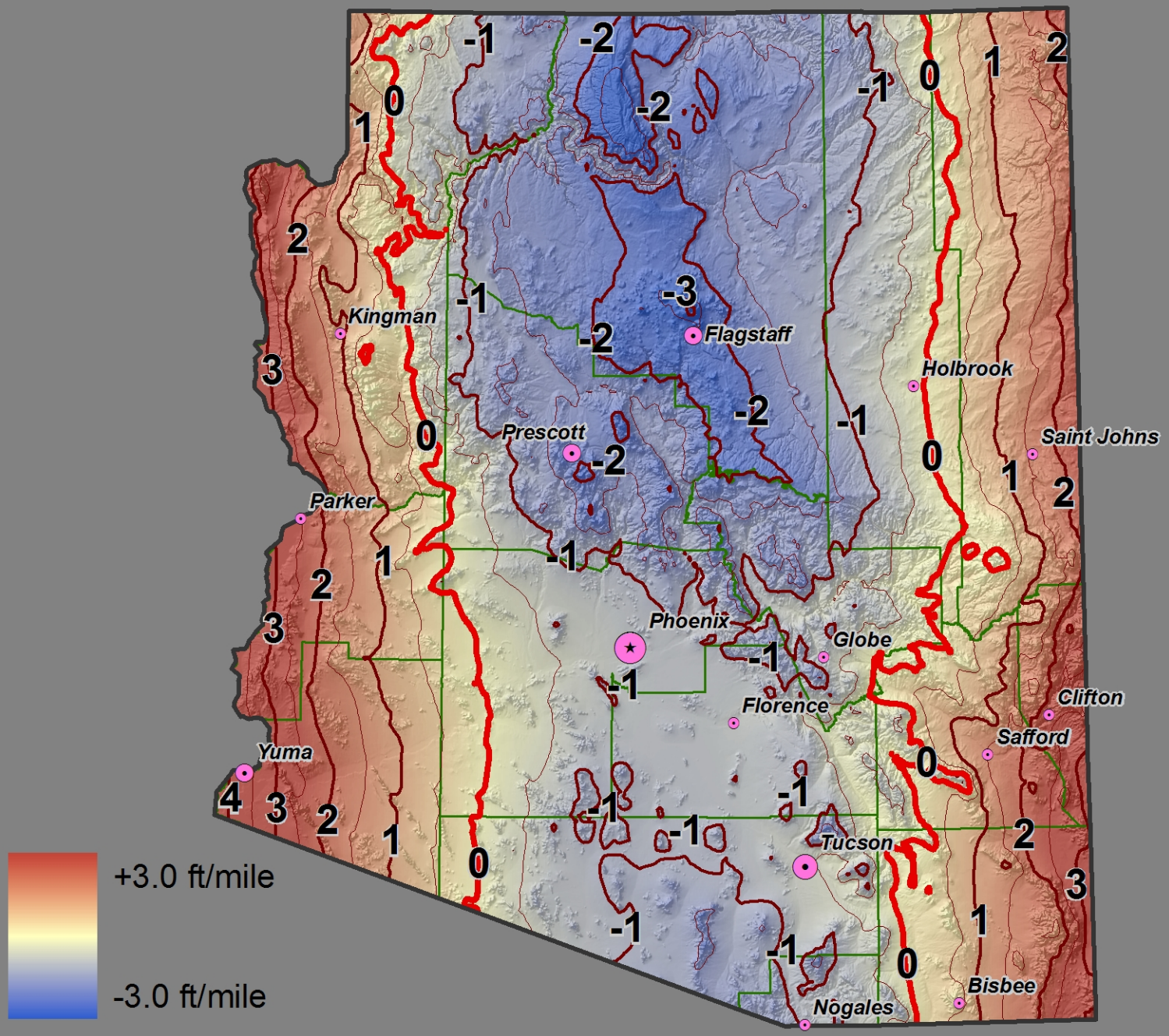
State Plane (AZ Zone C) linear distortion (feet/mile)



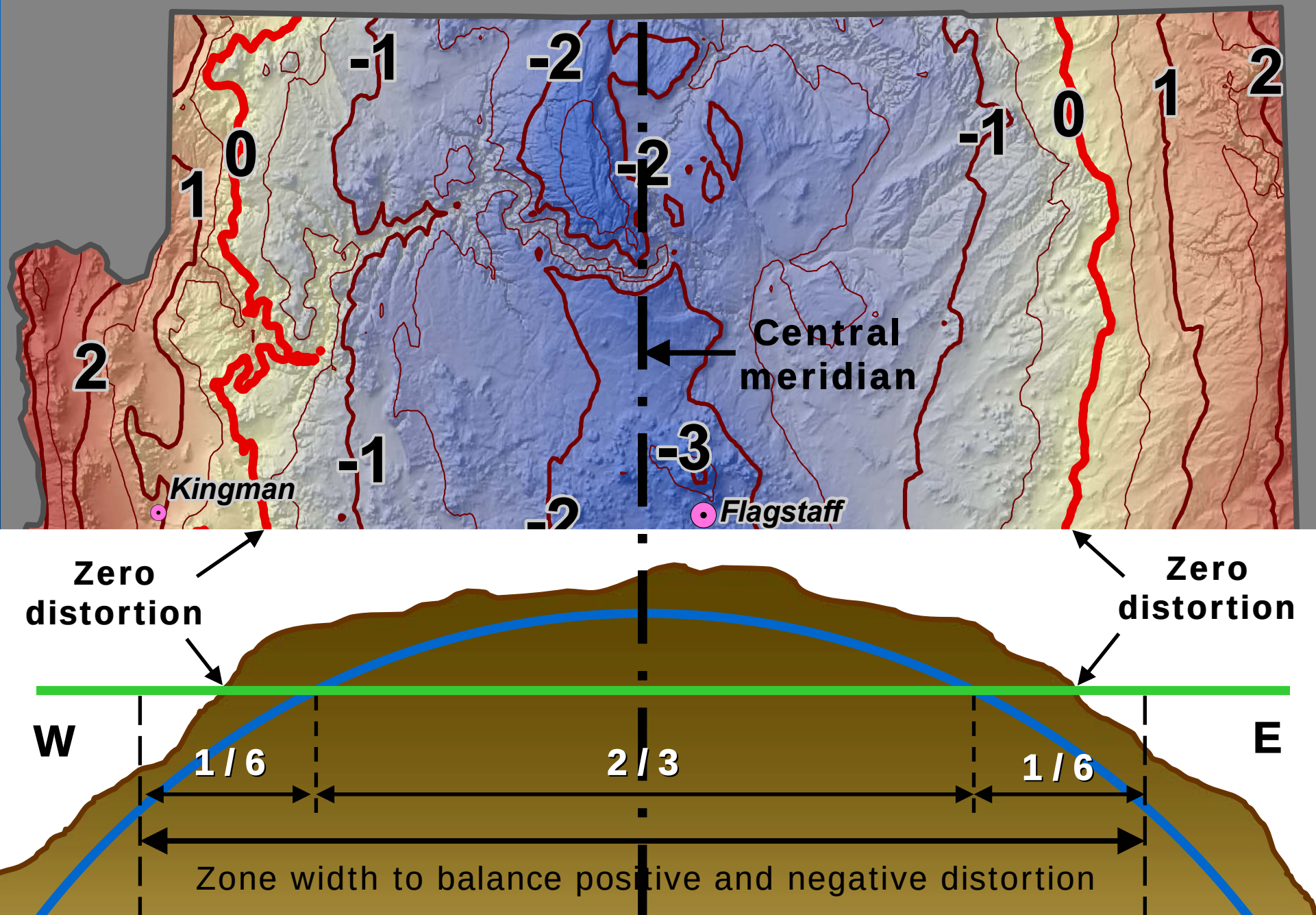
State Plane (AZ Zone C) linear distortion (parts per million)



State Plane (AZ Zone C) linear distortion (feet/mile)



State Plane (AZ zone C) “linear distortion” (feet/mile)



Methods

for design of low-distortion grid coordinate systems

1. Design projection for specific project area
   - *Will discuss this method later in presentation*

Methods

for design of low-distortion grid coordinate systems

1. Design projection for specific project area
2. Scaling the reference ellipsoid

   Problems:

   - Increases complexity with negligible reduction in distortion
   - More difficult to implement; requires new ellipsoid (datum) for every project area
   - *Can generate new set of latitudes that are different from the original latitudes* (can lead to confusion)
     - E.g., In Flagstaff, change can exceed 20 feet

Methods

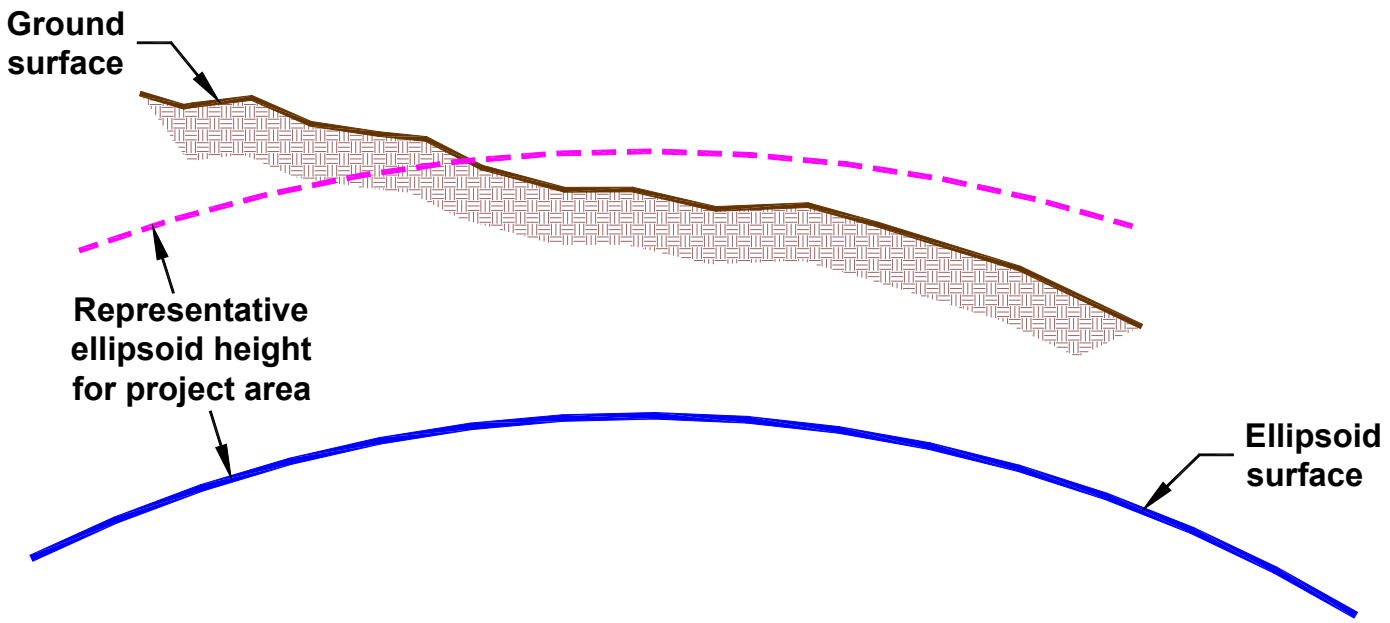
for design of low-distortion grid coordinate systems

1. Design projection for specific project area
2. Scaling the reference ellipsoid
3. Scaling existing projections (e.g., "modified" State Plane)

Problems:

- Generates coordinates with values similar to "true" State Plane (can lead to confusion)
  - Can eliminate this by translating grid coordinates
- More difficult to implement; often results in "messy" projection definition
  - Can reduce this problem through judicious selection of "scaling" parameters
- Does not reduce convergence angle
- **Most important: Does not minimize distortion over as large an area as the other methods**

Local grid coordinate system designed for specific project location, showing extent of low-distortion coverage



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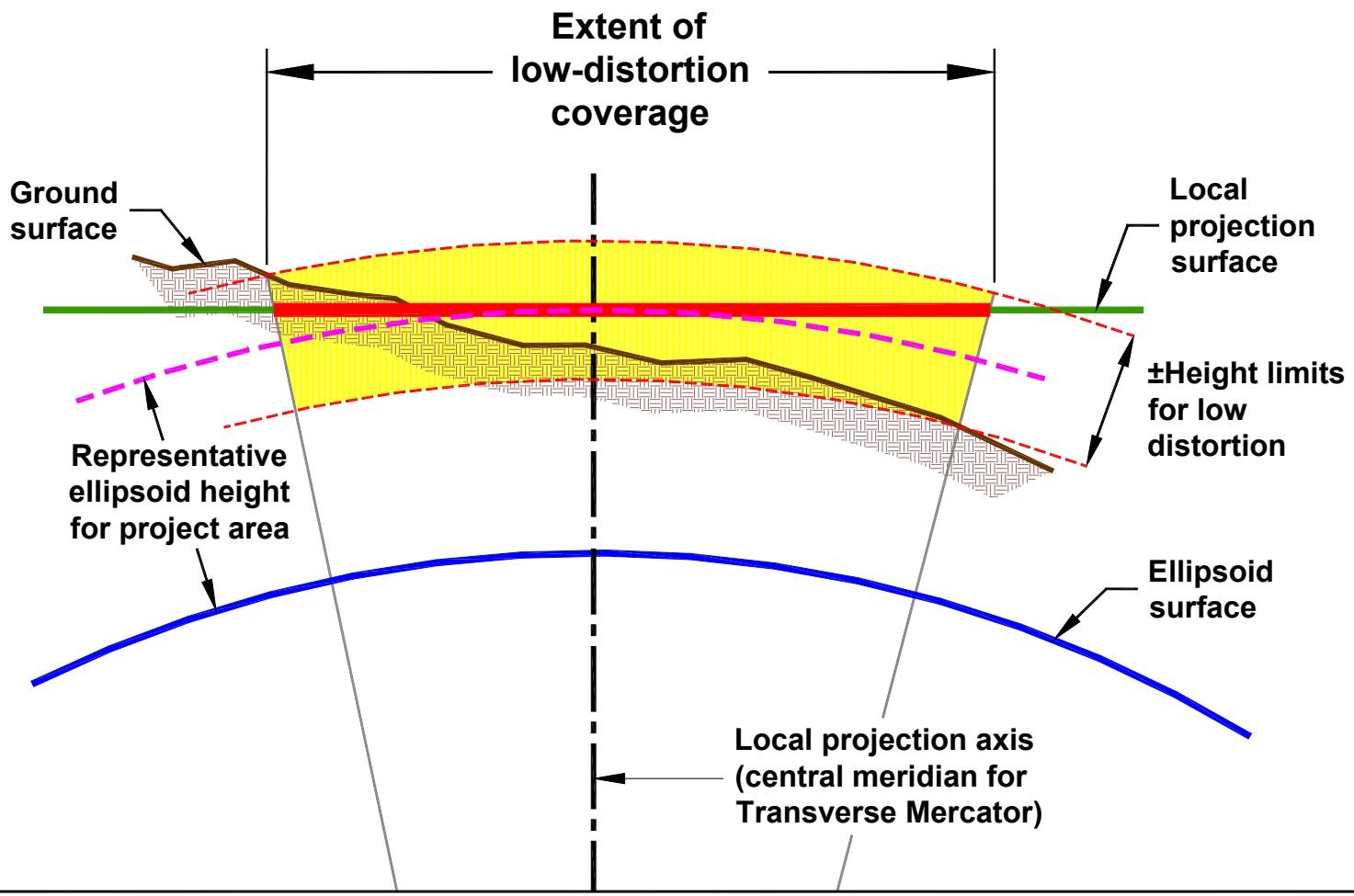
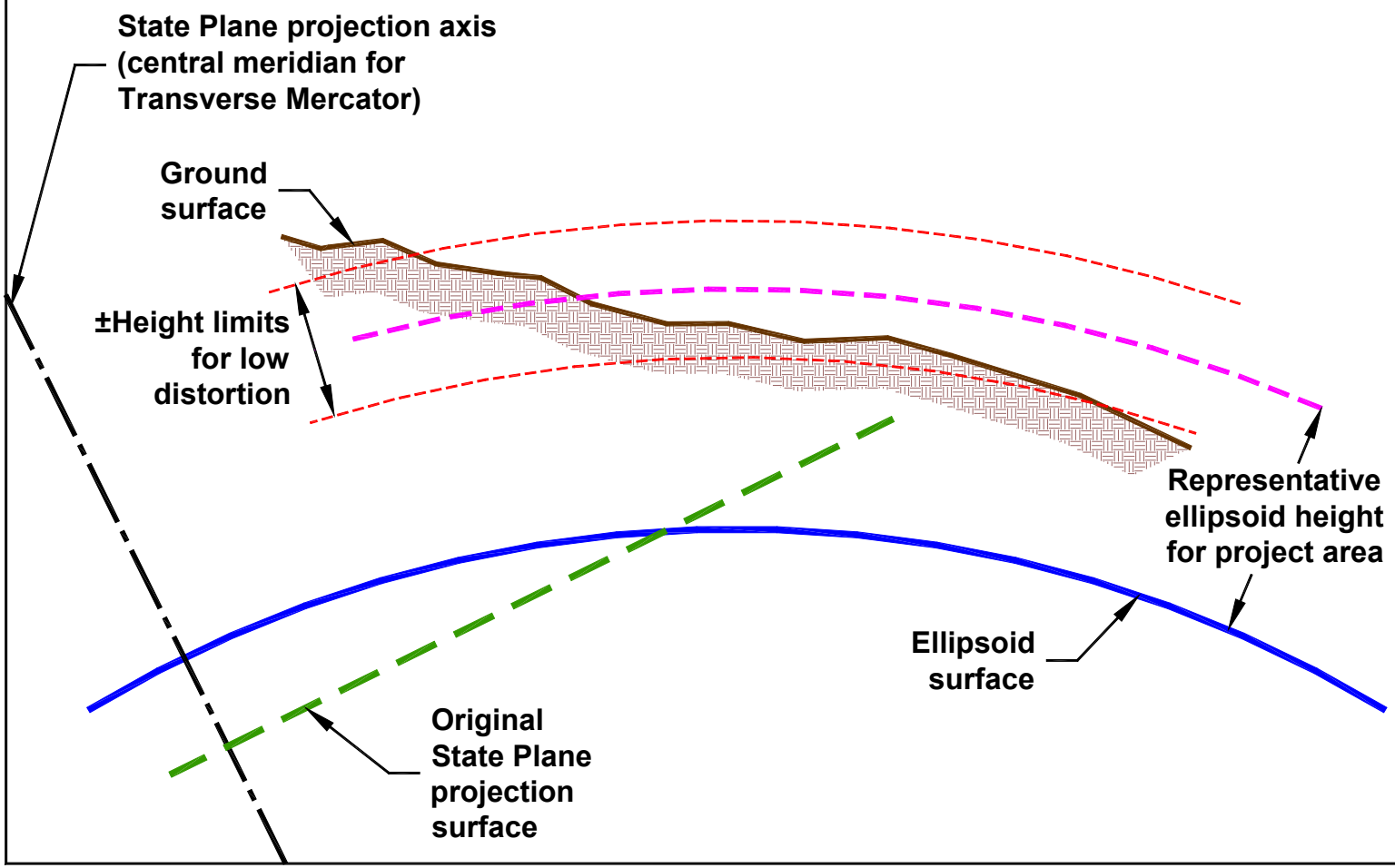


Local grid coordinate system based on "modified" State Plane approach, showing reduced extent of low-distortion coverage



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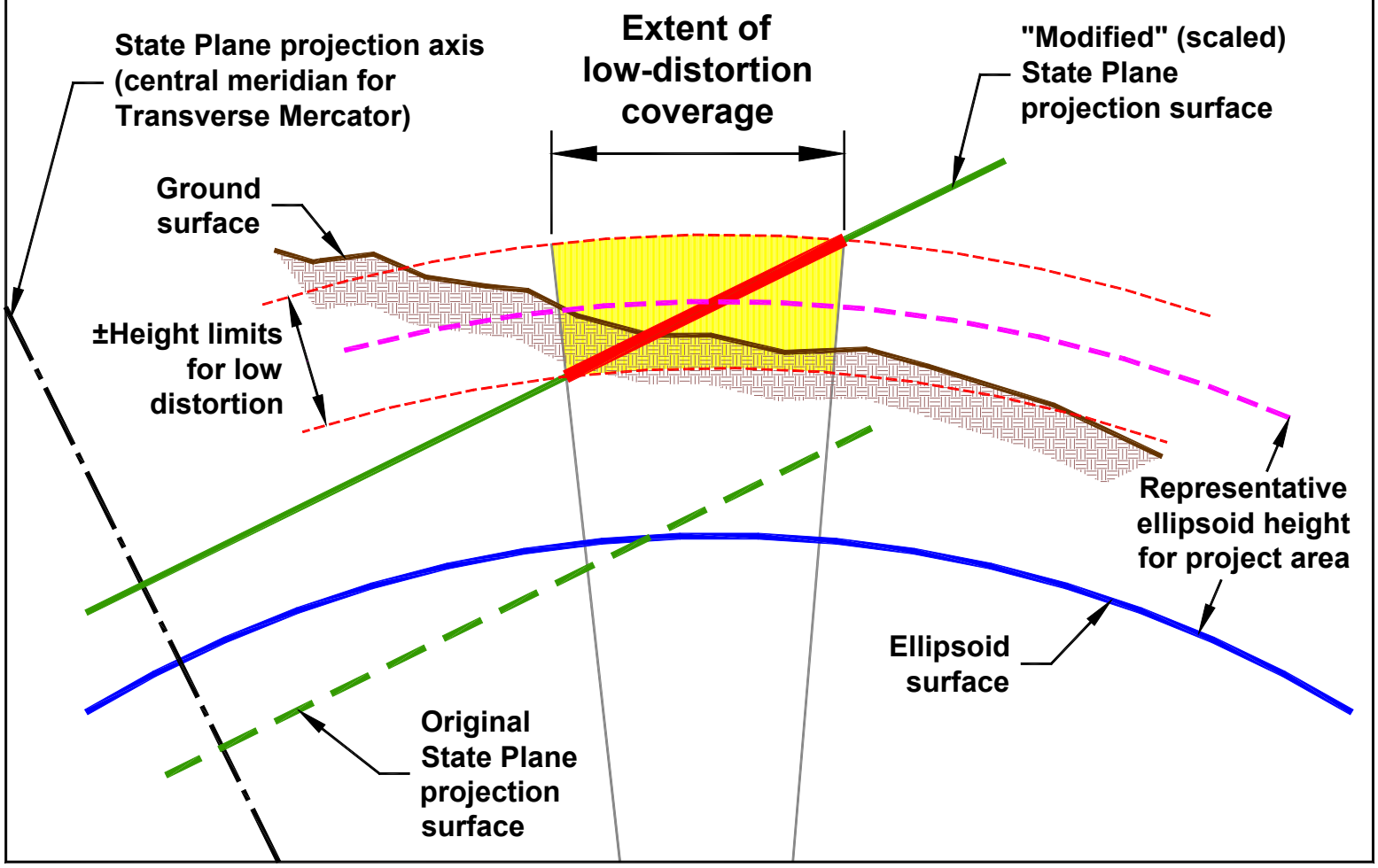
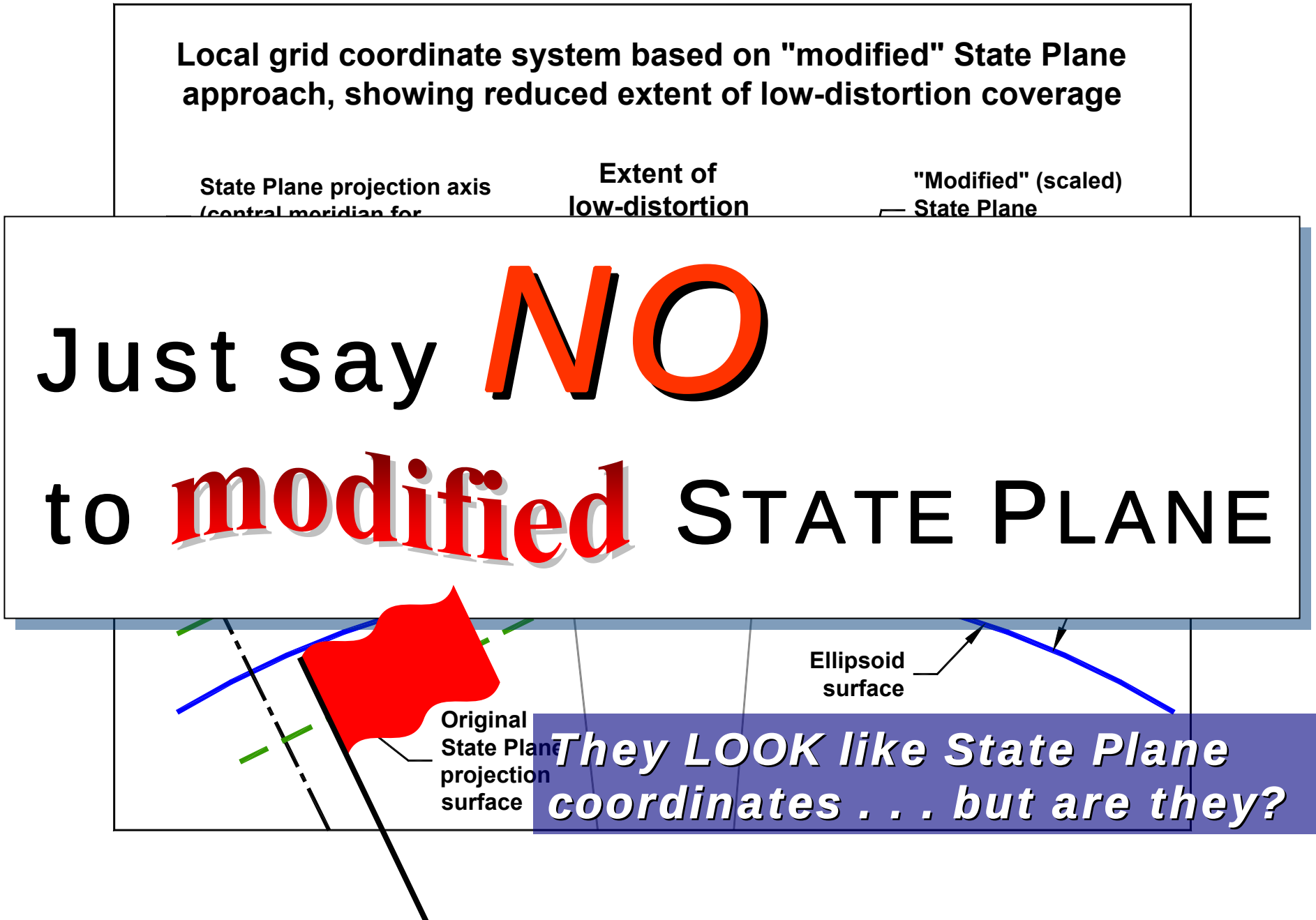



Local grid coordinate system based on "modified" State Plane approach, showing reduced extent of low-distortion coverage
State Plane projection axis
Extent of low-distortion
"Modified" (scaled) State Plane
Ellipsoid surface
Original State Plane projection surface
Just say NO to modified STATE PLANE
They LOOK like State Plane coordinates . . . but are they?

Six steps for designing a Low Distortion Projection (*see handout*)

1. Define project area and choose *representative* ellipsoid height, h_0 (not elevation)
   - The *average* height of an area may not be appropriate (e.g., if project is adjacent to a mountain)
2. Place central meridian near center of project area
3. Compute projection axis scale as:

$$k_0 = 1 + \frac{h_0}{R}$$

   - Where R is Earth radius
   - Can use 20,920,000 feet for Oregon, or can compute for specific latitude — k_0 value is refined in next step
   - For Transverse Mercator projection axis is the central meridian; for Lambert Conformal Conic projection axis is the standard parallel

Six steps for designing a Low Distortion Projection (*see handout*)

4. Check distortion throughout project area
   - Best approach: Compute distortion over entire area, generate distortion contours, and refine k_0
   - Compute distortion as:

$$\delta = k\left(\frac{R}{R+h}\right) - 1$$

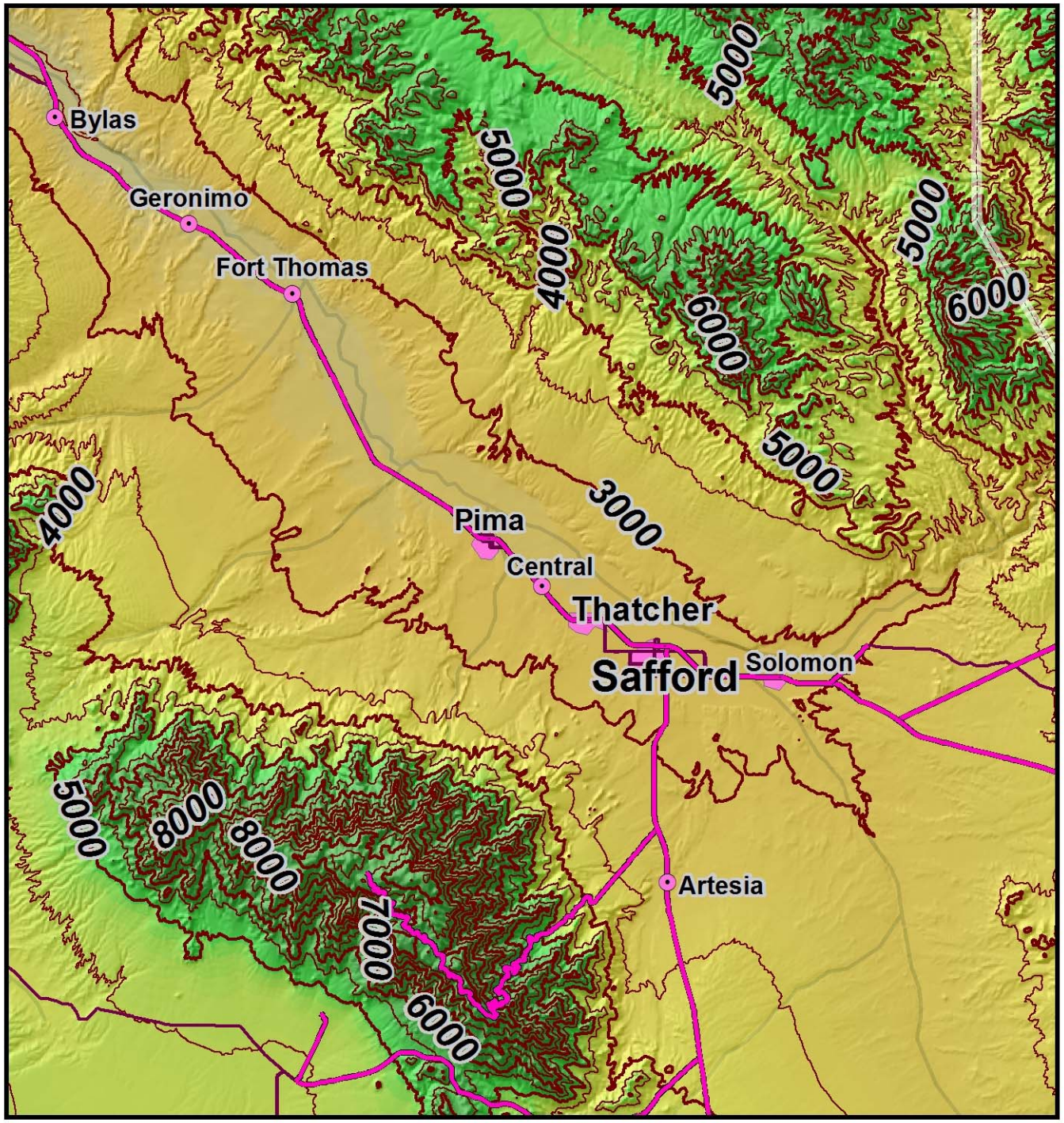
Combined Scale Factor

     - Multiply δ by 1,000,000 to get distortion in ppm

5. Keep definition *simple* and *clean*
   - Define k_0 to no more than **SIX** decimal places
   - Geodetic origins to nearest arc-minute
   - Grid origins using whole numbers, small as possible

6. Explicitly define *linear unit* and *geodetic datum*
   - E.g., International foot, NAD 83 (2007)

Gila Valley Topographic Height

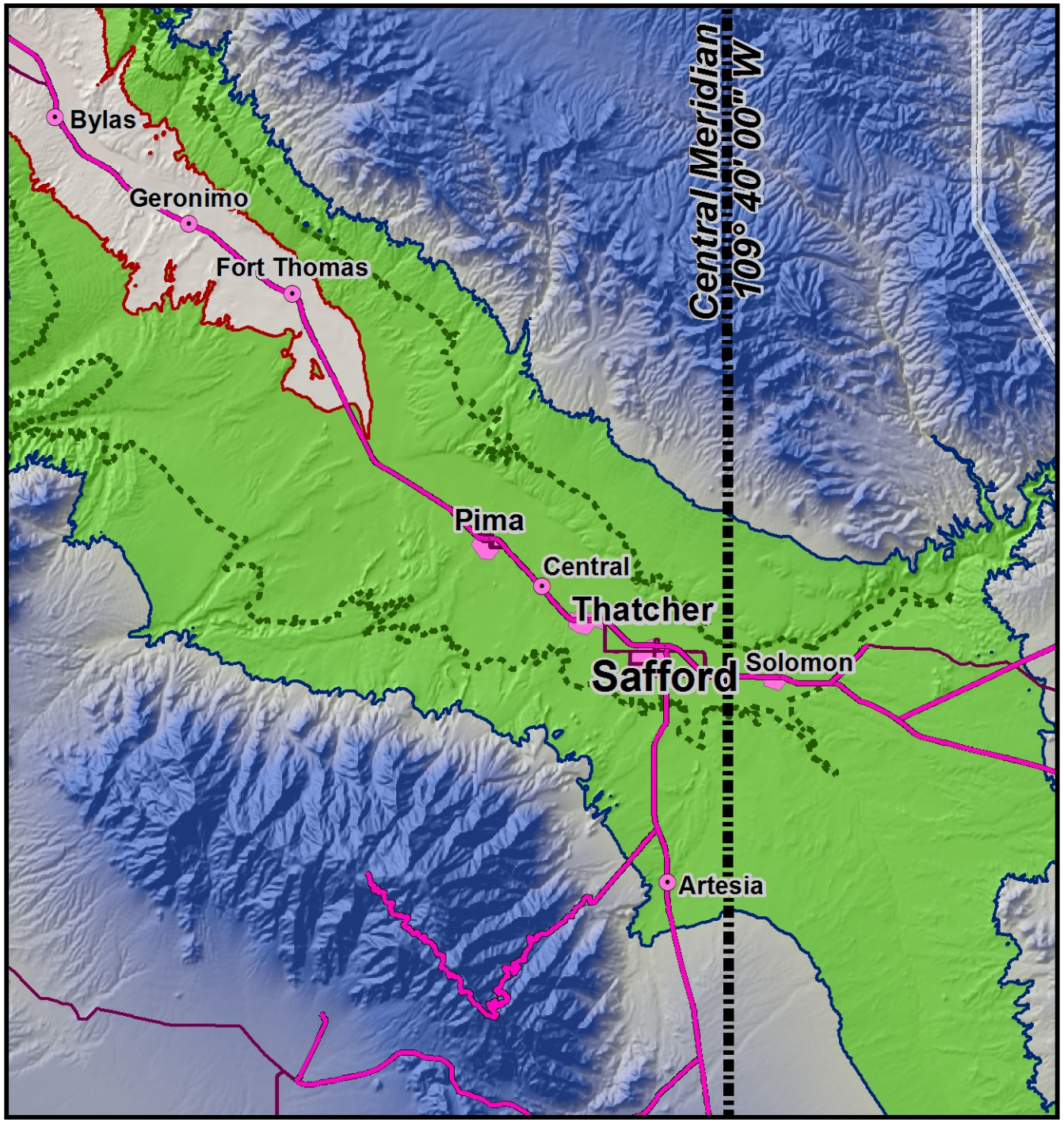


0 5 10 Miles

Ellipsoidal height contours

1000-ft contours

500-ft contours



Gila Valley Low Distortion Projection

Design alternatives

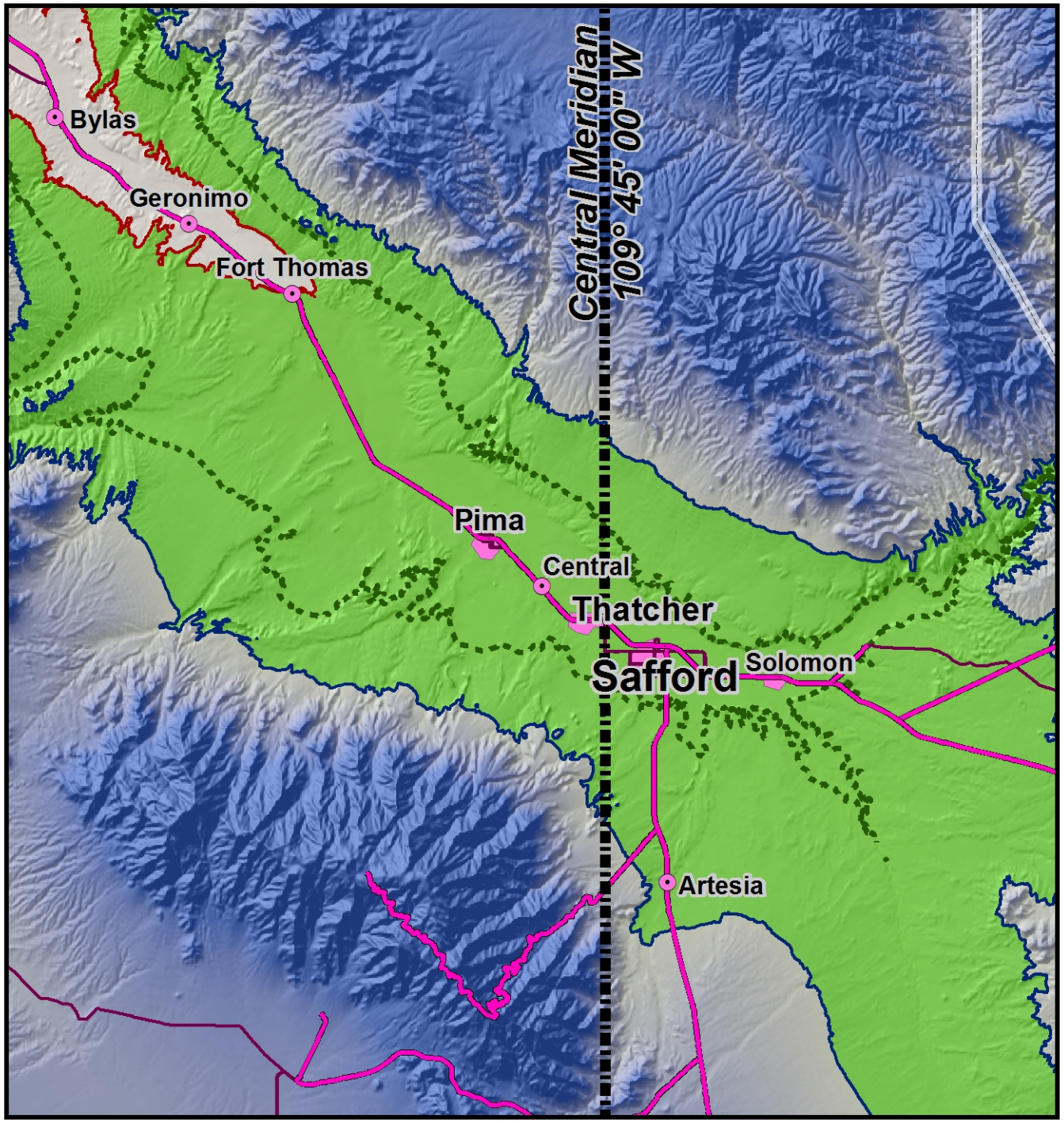
Transverse Mercator

CM = 109° 40' 00" W
CM scale = 1.000140

0 5 10 Miles

Linear distortion

- +20 ppm (+0.1 ft/mile)
- Zero distortion
- -20 ppm (-0.1 ft/mile)
- ±20 ppm (±0.1 ft/mile)
- +100 ppm
- -100 ppm



Gila Valley Low Distortion Projection

Design alternatives

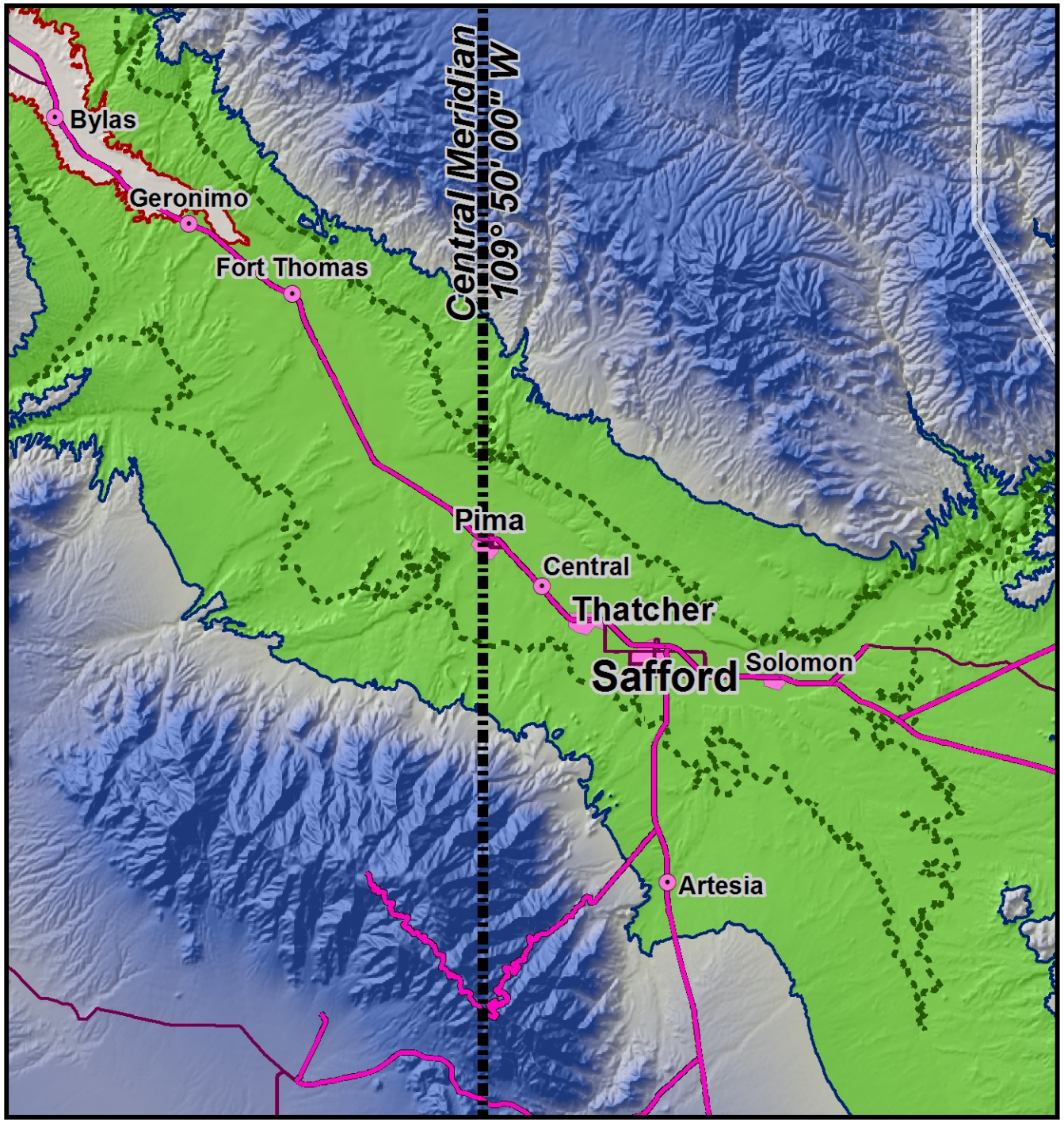
Transverse Mercator

CM = 109° 45' 00" W
CM scale = 1.000140

0 5 10 Miles

Linear distortion

- +20 ppm (+0.1 ft/mile)
- Zero distortion
- -20 ppm (-0.1 ft/mile)
- ±20 ppm (±0.1 ft/mile)
- +100 ppm
- -100 ppm



Gila Valley Low Distortion Projection

Design alternatives

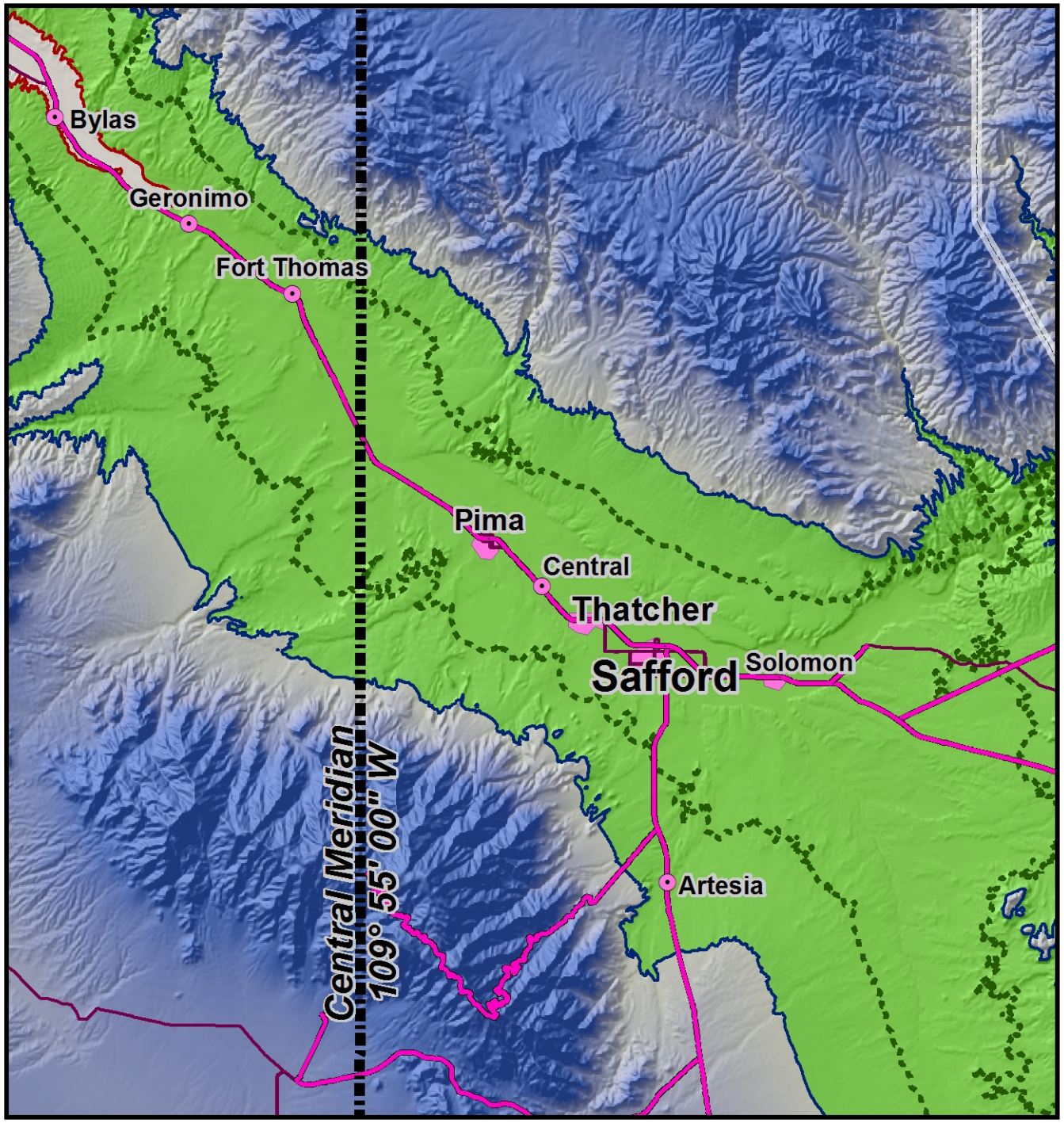
Transverse Mercator

CM = 109° 50' 00" W
CM scale = 1.000140

0 5 10 Miles

Linear distortion

- +20 ppm (+0.1 ft/mile)
- Zero distortion
- -20 ppm (-0.1 ft/mile)
- ±20 ppm (±0.1 ft/mile)
- +100 ppm
- -100 ppm



Gila Valley Low Distortion Projection

Design alternatives

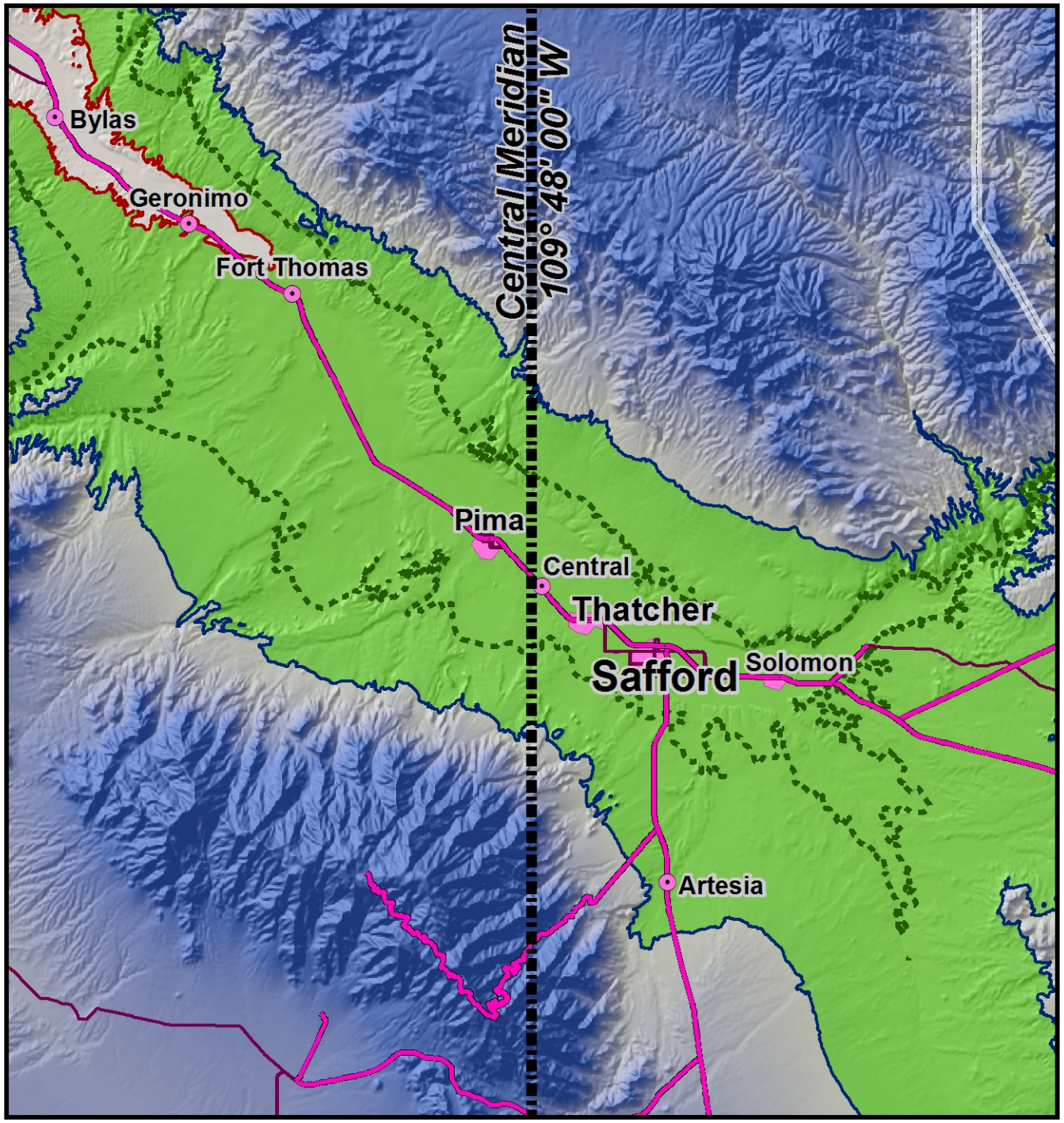
Transverse Mercator

CM = 109° 55' 00" W
CM scale = 1.000140

0 5 10 Miles

Linear distortion

- +20 ppm (+0.1 ft/mile)
- Zero distortion
- -20 ppm (-0.1 ft/mile)
- ±20 ppm (±0.1 ft/mile)
- +100 ppm
- -100 ppm



Gila Valley Low Distortion Projection

Design alternatives

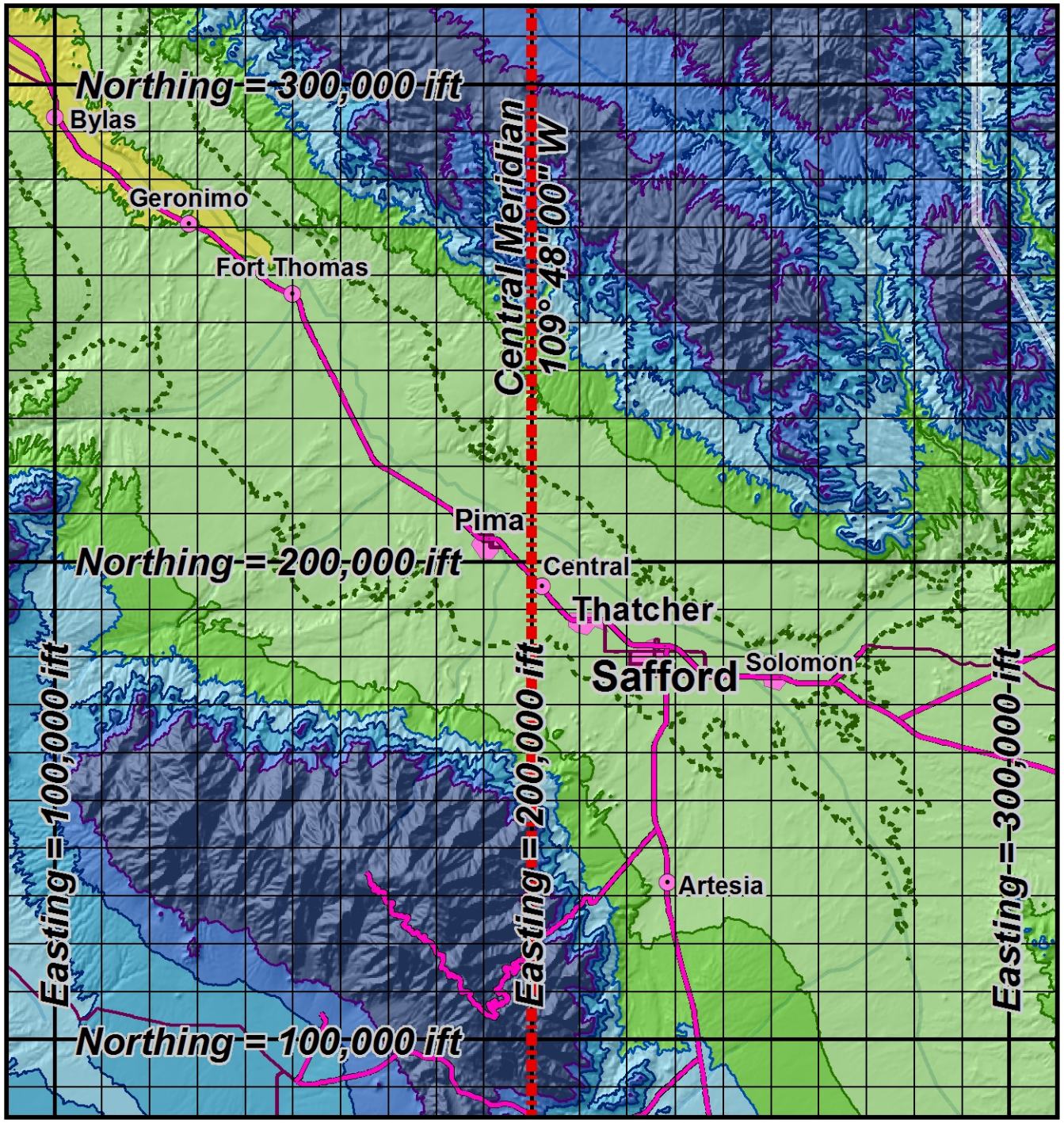
Transverse Mercator

CM = 109° 48' 00" W
CM scale = 1.000140

0 5 10 Miles

Linear distortion

- +20 ppm (+0.1 ft/mile)
- Zero distortion
- -20 ppm (-0.1 ft/mile)
- ±20 ppm (±0.1 ft/mile)
- +100 ppm
- -100 ppm



Gila Valley Low Distortion Projection

Linear unit: International foot
North American Datum of 1983
Transverse Mercator projection

Latitude of origin: 32° 20' 00" N
Central meridian: 109° 48' 00" W
False northing: 0.000 ift
False easting: 200,000.000 ift
CM scale factor: 1.00014 (exact)

0 5 10 Miles

Linear distortion

- Zero distortion
- < -100 ppm (< -0.5 ft/mile)
- -100 to -80 ppm
- -80 to -60 ppm
- -60 to -40 ppm
- -40 to -20 ppm
- -20 to 20 ppm (±0.1 ft/mile)
- 20 to 40 ppm
- 40 to 60 ppm
- 60 to 80 ppm
- 80 to 100 ppm
- > 100 ppm (> 0.5 ft/mile)

Gila Valley, Arizona
Low Distortion Projection

Projection parameters	Low Distortion Projection (Transverse Mercator)	"Equivalent" modified SPCS 83 AZ E
Origin latitude	**32°20'00"N**	31°00'00"N
Central meridian longitude	**109°48'00"W**	110°10'00"W
False northing	**0.000 ift**	0.000 ift
False easting	**200,000.000 ift**	700,160.341 168 614...ift
Central meridian scale	**1.00014 (exact)**	1.000 129 035 906 42...

Spatial Data Requirements

Include in surveying & mapping services contracts

- Completely define the coordinate system
  - Linear unit (meter, foot, what kind of foot?)
  - Geodetic and vertical datums
  - Map projection parameters
    - *Is "distortion" an issue for your project . . . ?*
- Require ties to ***published*** control
  - Best source for control: National Geodetic Survey (NGS)
    - But control must be more accurate than surveying or mapping method used

A project that requires ties to "U.S. Coast & Geodetic Survey" control

Spatial Data Requirements

Include in surveying & mapping services contracts

- Specify ***accuracy*** requirements
  - Use objective, robust, defensible methods
    - Make use of published standards (e.g., NSSDA)
    - Least-squares network adjustment (but no cheating!)
  - *Remember:* Accuracy may not be same as precision
- Documentation is ***essential*** (metadata!)
  - Report methods, procedures, and results for creating final deliverables
  - *Must* include any coordinate transformations
    - E.g., datum transformations, “rubber sheeting”

Documentation (metadata!)

- If a custom coordinate system is used, it ***MUST*** be documented
  - This is essential for correctly incorporating data into a GIS
    - Should *never* have to resort to approximate methods (e.g., “rubber sheeting”)
  - Consistent with spirit of “leaving footsteps…”
    - Others should be able to retrace the original surveying or mapping product (*without* having to contact the person who created it!)

Example of typical survey metadata . . .

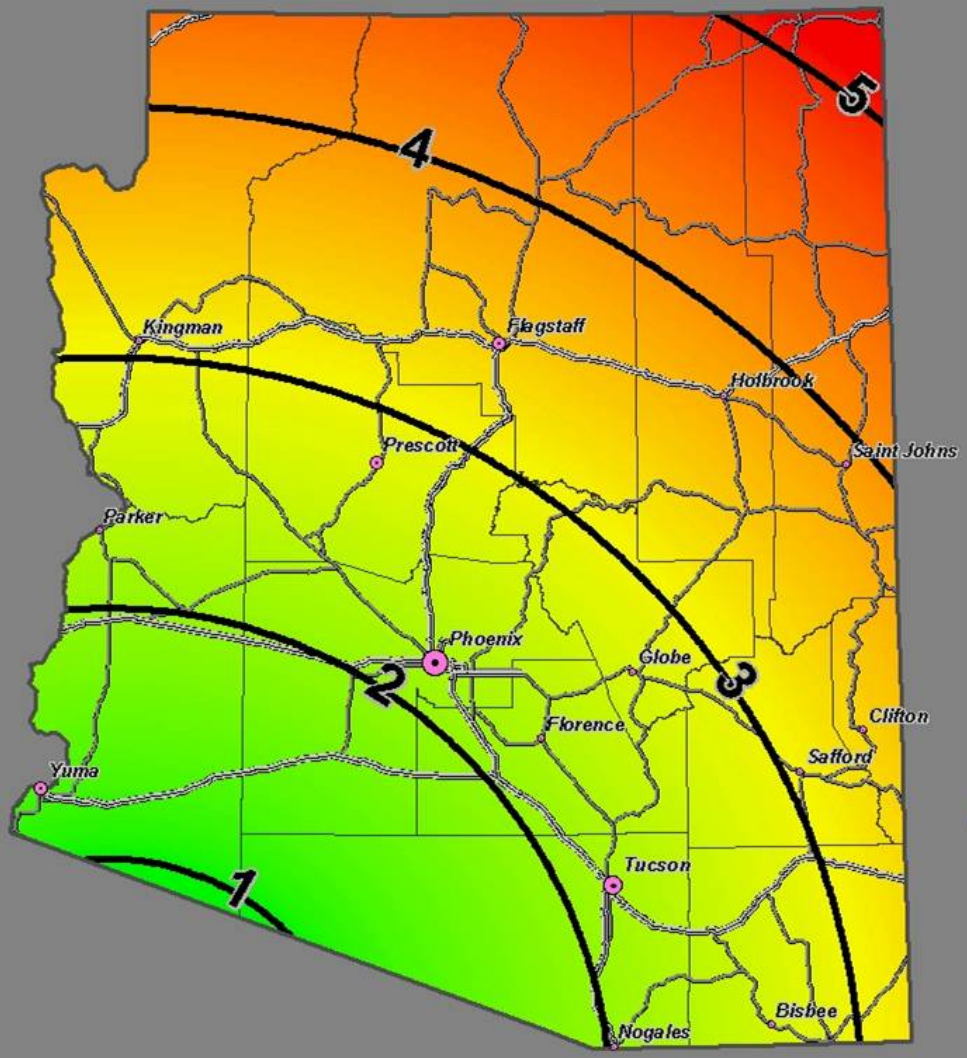
Example of survey metadata (*see handout*)

Basis of Bearings and Coordinates

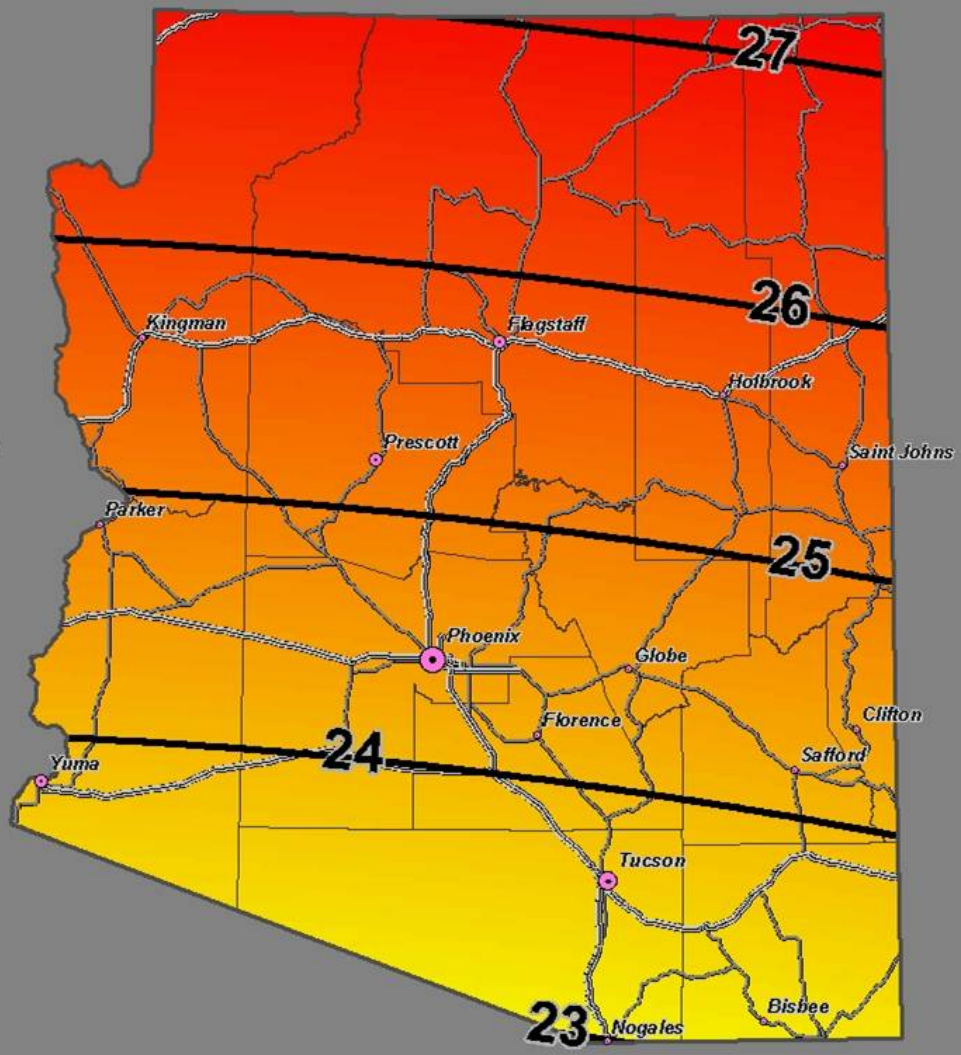
Linear unit: International foot (ift)

Horizontal difference in coordinates (in feet) due to difference between International and US Survey feet

SPCS 83 AZ Central



UTM 83 12 North



Example of survey metadata (*see handout*)

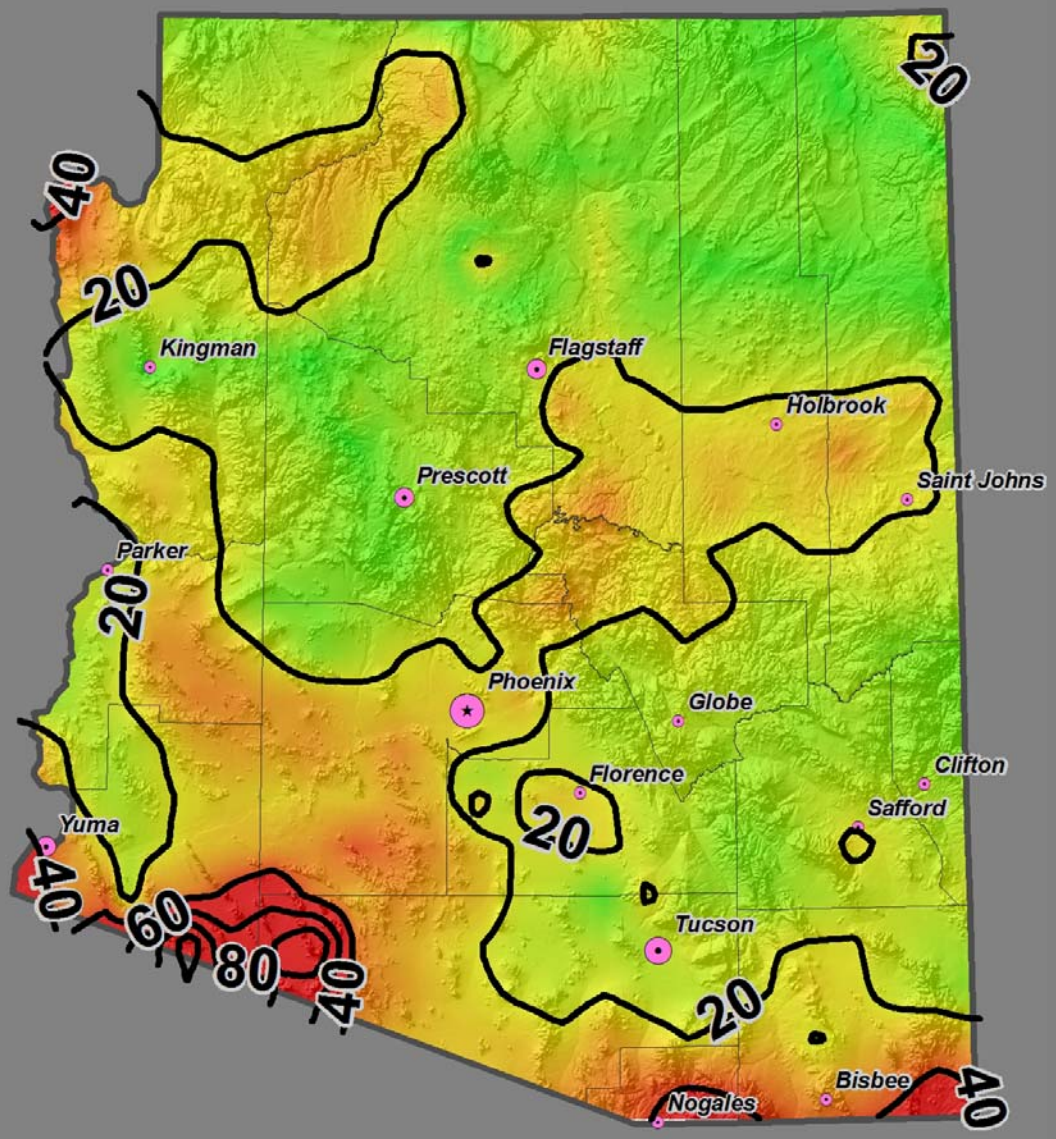
Basis of Bearings and Coordinates

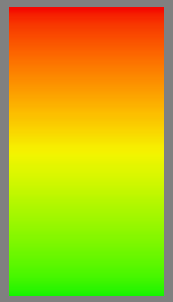
Linear unit: International foot (ift)

Geodetic datum: North American Datum of 1983 (1992)

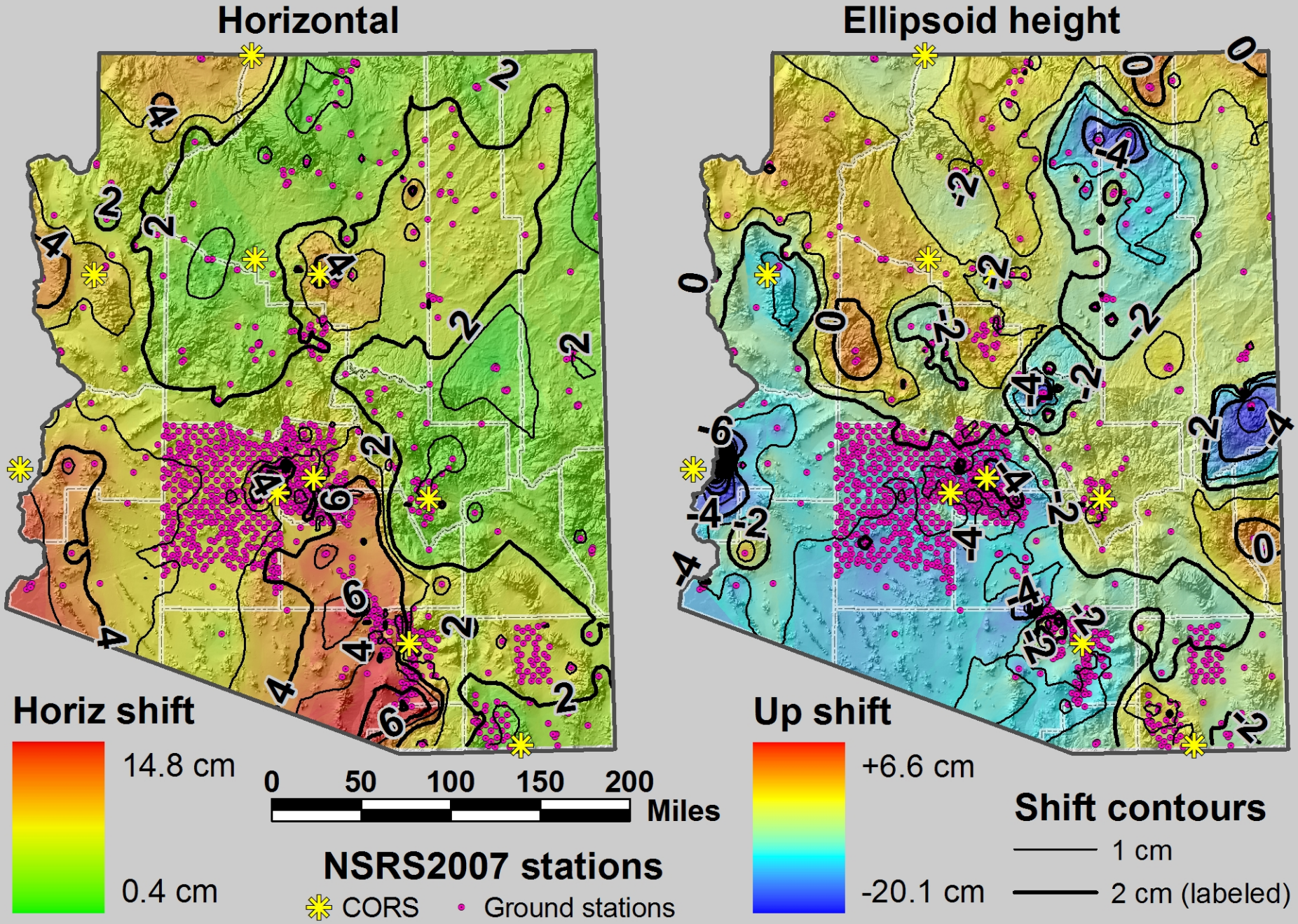
Horizontal difference between NAD 83 flavors (centimeters)

NAD 83 (1986) and (1992)



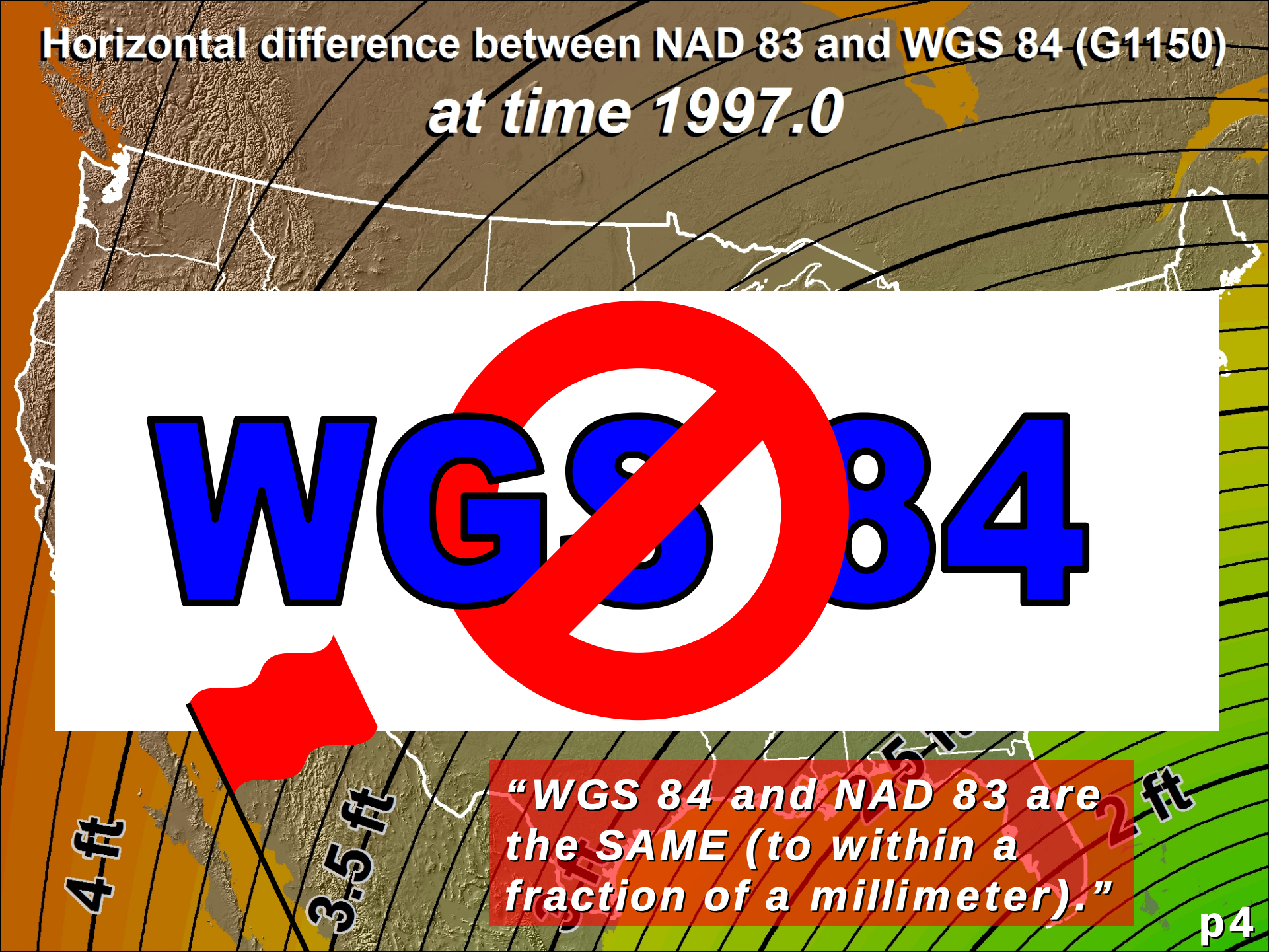


NSRS2007 national readjustment coordinate change (cm)



Horizontal difference between NAD 83 and WGS 84 (G1150)

at time 1997.0



"WGS 84 and NAD 83 are the SAME (to within a fraction of a millimeter)."

Example of survey metadata (*see handout*)

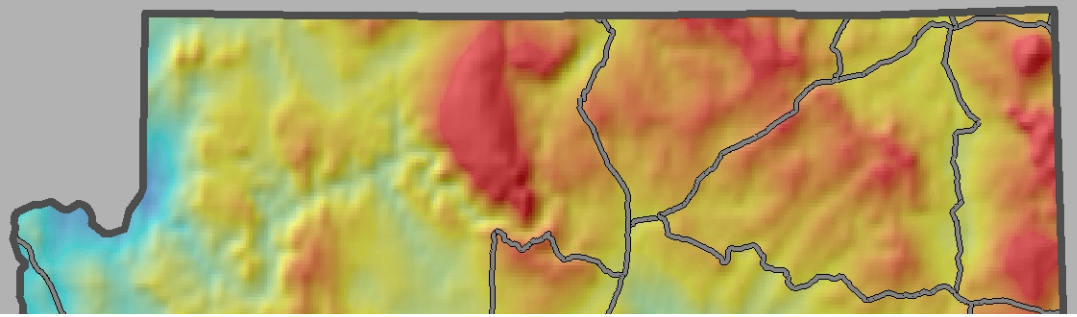
Basis of Bearings and Coordinates

Linear unit: International foot (ift)

Geodetic datum: North American Datum of 1983 (1992)

Vertical datum: North American Vertical Datum of 1988 (see below)

VERTCON vertical difference, NAVD 88 *minus* NGVD 29 (feet)



Iulhqgv...

don't let iulhqgv...

use *NGVD 29*

Elevation reference specified as "Mean Sea Level" datum

Example of survey metadata (*see handout*)

Basis of Bearings and Coordinates

Linear unit: International foot (ift)

Geodetic datum: North American Datum of 1983 (1992)

Vertical datum: North American Vertical Datum of 1988 (see below)

System: Arizona LDP

Zone: Gila Valley

Projection: Transverse Mercator

Latitude of grid origin: 32° 20’ 00” N

Longitude of central meridian: 109° 48’ 00” W

Northing at grid origin: 0.000 ft

Easting at central meridian: 200,000.000 ft

Scale factor on central meridian: 1.00014 (exact)

All distances and bearings shown hereon are grid values based on the preceding projection definition. The projection was defined such that grid distances are equivalent to “ground” distances in the project area.

The basis of bearings is geodetic north. Note that the grid bearings shown hereon (or implied by grid coordinates) do not equal geodetic bearings due to meridian convergence.

Example of survey metadata (*see handout*)

Orthometric heights (elevations) were transferred to the site from NGS control station “P 439” (PID CY0725) using GPS with NGS geoid model “GEOID03” referenced to the current published NAVD 88 height of this station (889.460 m).

The survey was conducted using GPS referenced to the National Spatial Reference System. A partial list of point coordinates is given below (additional coordinates are available upon request). Local network accuracy estimates are given at the 95% confidence level and are based on an appropriately constrained least-squares adjustment of over-determined and statistically independent observations.

Point #1 “SAFFORD BASE ARP”, permanent GPS base (off site)

		Estimated accuracy
Latitude = 32°48’07.31561”N	Northing = 170,564.00 ift	Horiz = Fixed
Longitude = 109°42'42.84664”W	Easting = 227,075.29 ift	Vert = Fixed
Ellipsoidal height: 2945.42 ift	Elevation = 3033.83 ift	

Point #1002, ½” rebar with cap, derived coordinates (on site)

		Estimated accuracy
Latitude = 32°50'06.81662"N	Northing = 182,643.21 ift	Horiz = ±0.034 ift
Longitude = 109°42'47.90144"W	Easting = 226,633.86 ift	Vert = ±0.056 ift
Ellipsoidal height = 2822.41 ift	Elevation = 2910.73 ift	

Point #1006, ½” rebar with cap, derived coordinates (on site)

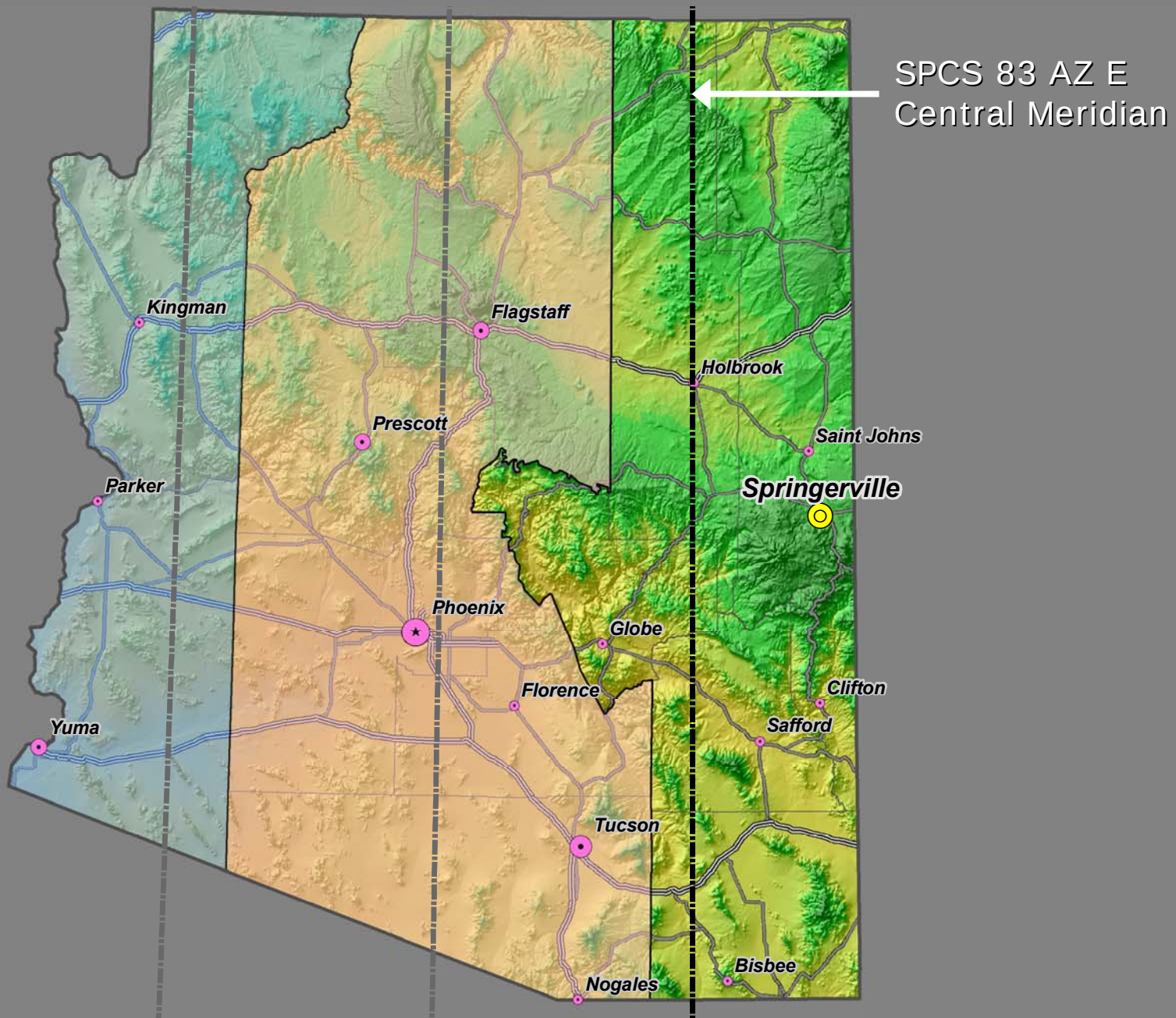
		Estimated accuracy
Latitude = 32°50'16.89645" N	Northing = 183,662.12 ift	Horiz = ±0.047 ift
Longitude = 109°42'47.93756"W	Easting = 226,629.94 ift	Vert = ±0.068 ift
Ellipsoidal height = 2815.73 ift	Elevation = 2904.04 ift	

Conclusions

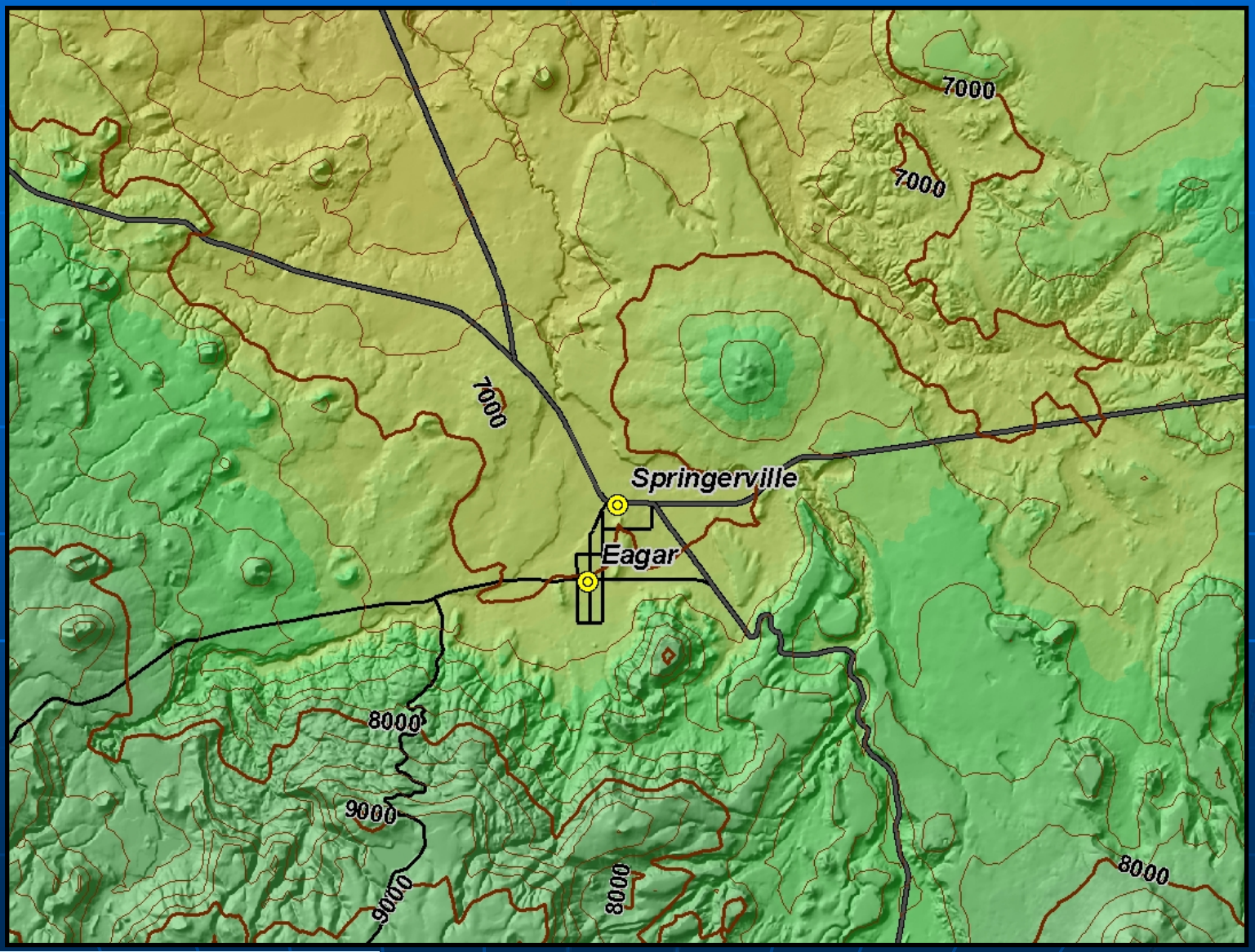
- Low Distortion Projections
  - A standardized method for ***minimizing*** distortion over ***largest*** area possible
    - Performance and simplicity superior to other methods
  - Satisfies needs of both the GIS and surveying communities, and both GPS and terrestrial
  - Enables direct use of survey data in a GIS
  - Reduces proliferation of local systems
    - Provides alternative for those who want (or need) “ground” coordinates
- Documentation (metadata) is ***essential***
  - Facilitates data transferability
    - Local data would be readily available to others (e.g., emergency services, Homeland Security)

Questions?

Example: Springerville (SPCS 83 AZ E)

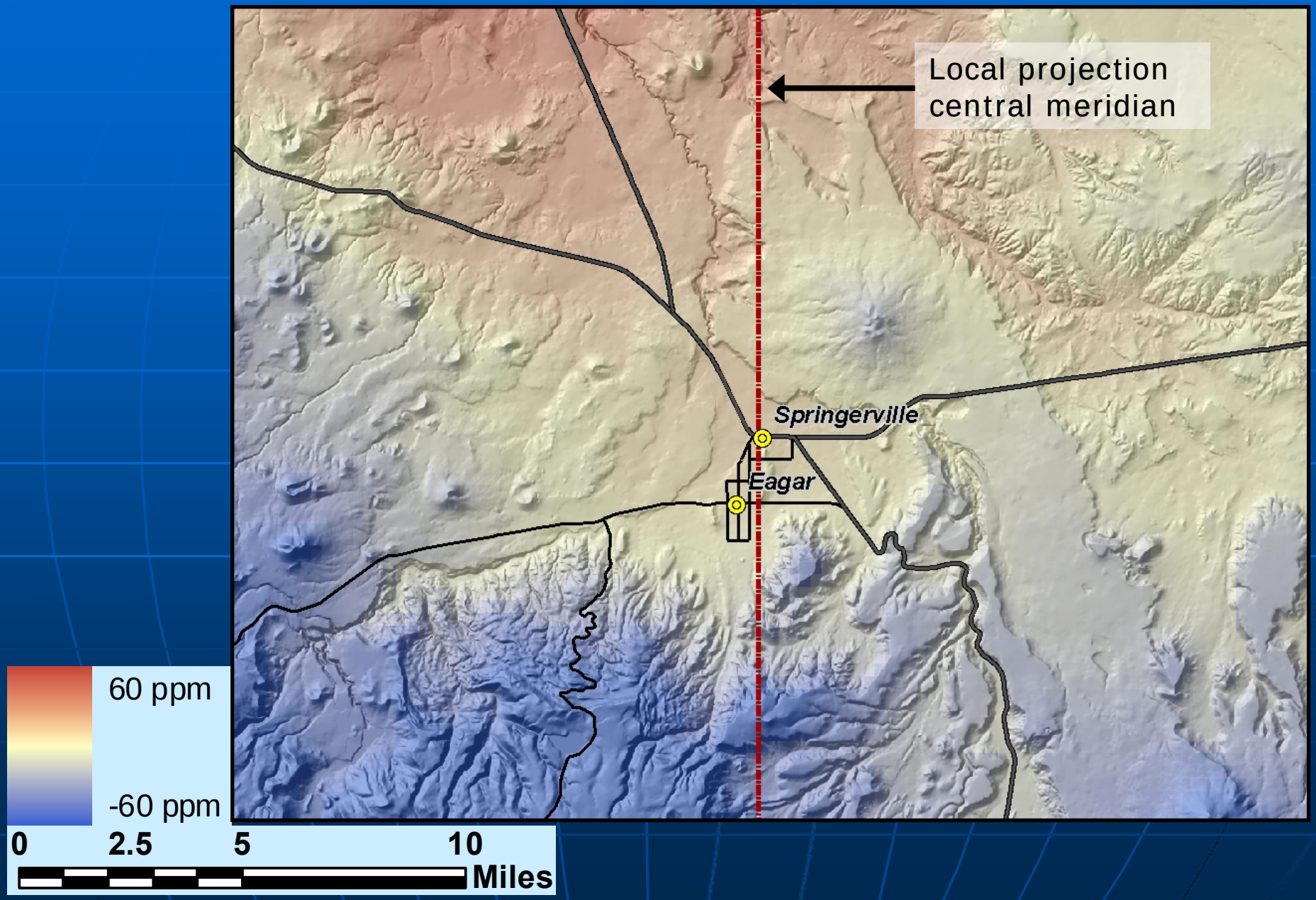


Ellipsoid height contours (feet)

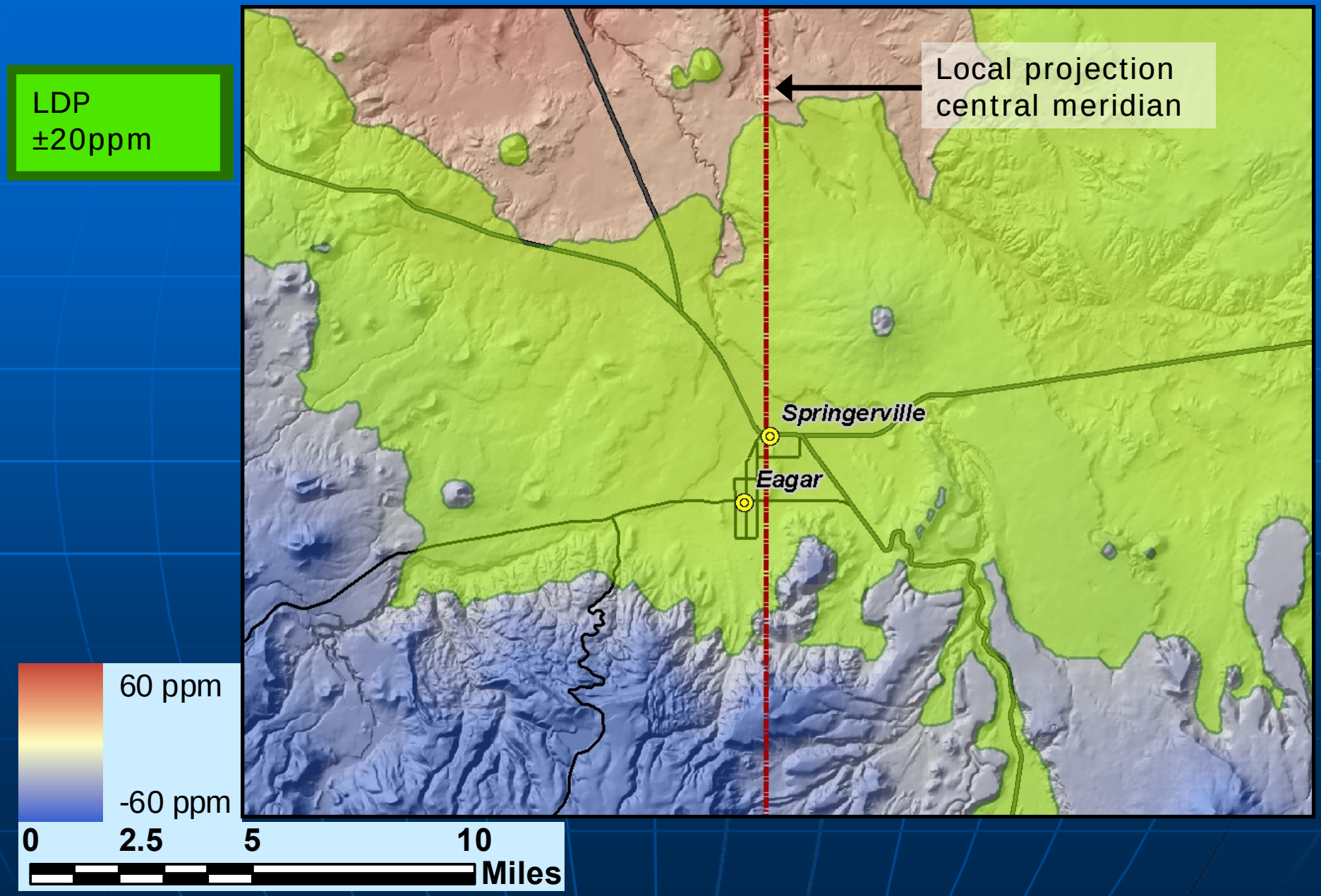


0 2.5 5 10 Miles

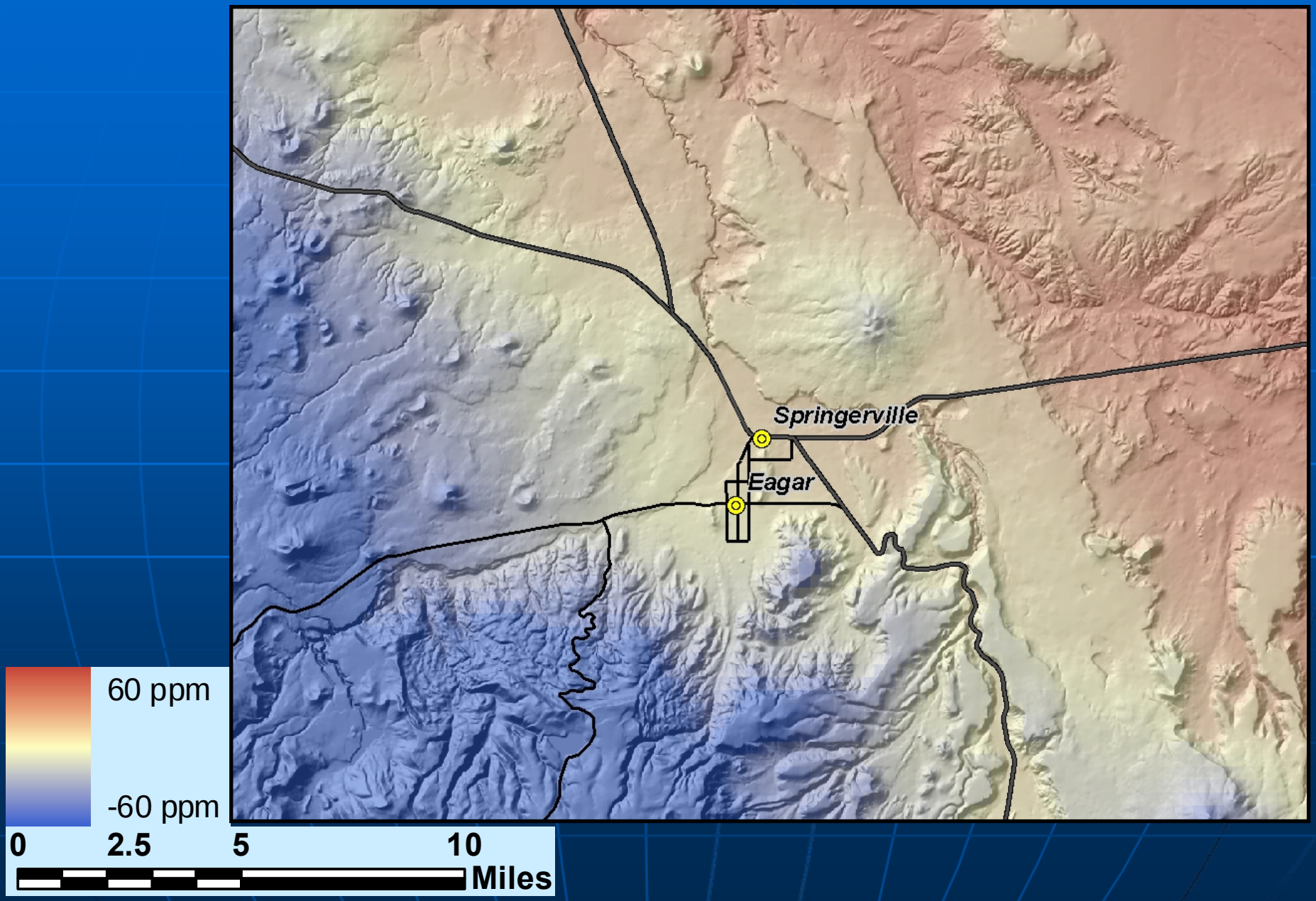
Linear distortion (local projection)



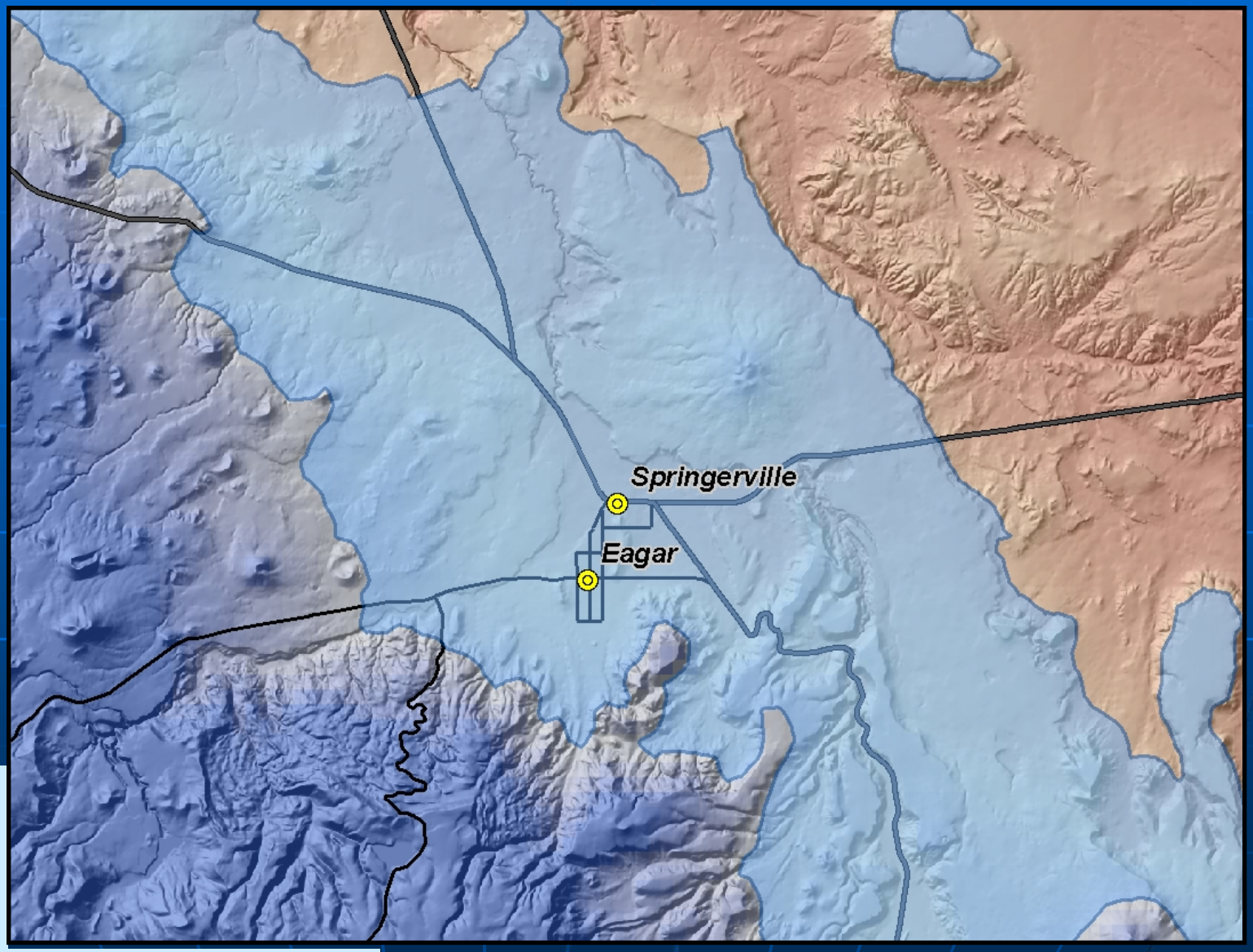
Linear distortion, ±20 ppm coverage (local projection)



Linear distortion (modified SPCS E)



Linear distortion, ±20 ppm coverage (modified SPCS E)



Mod SPCS E
±20ppm

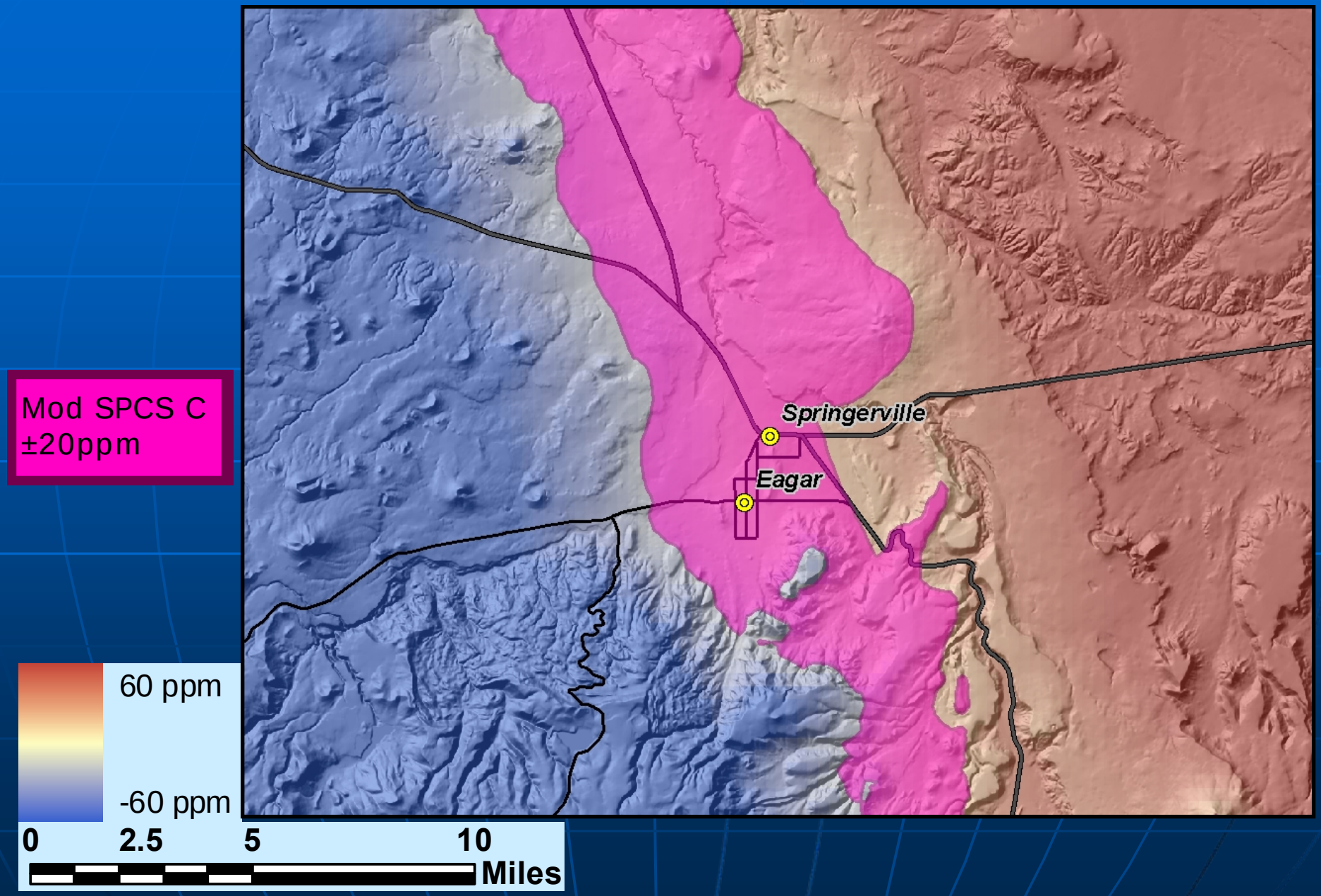
60 ppm

-60 ppm

0 2.5 5 10

Miles

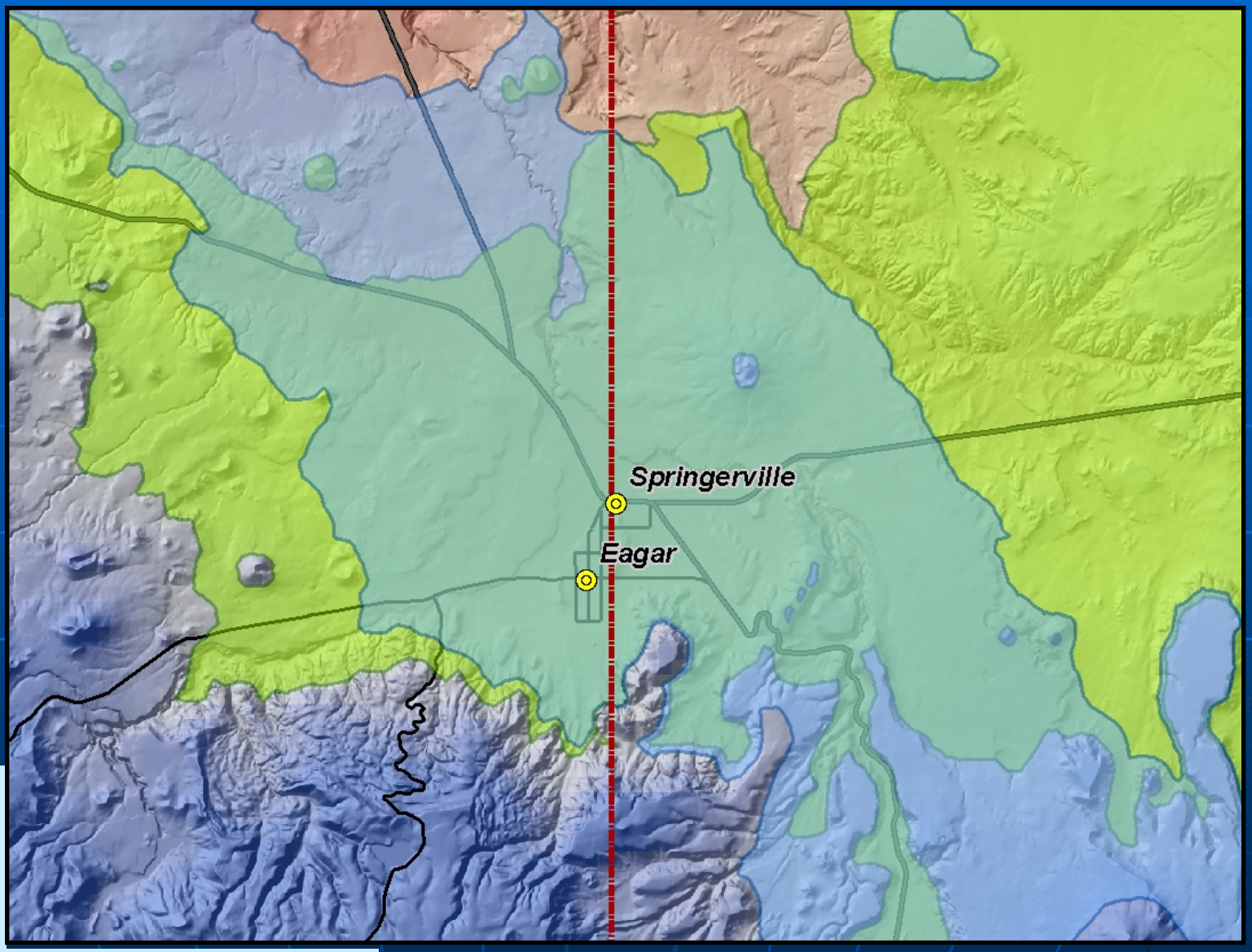
Linear distortion, ±20 ppm coverage (modified SPCS C)



Linear distortion, ±20 ppm coverage comparison

LDP
±20ppm

Mod SPCS E
±20ppm



60 ppm

-60 ppm

0 2.5 5 10

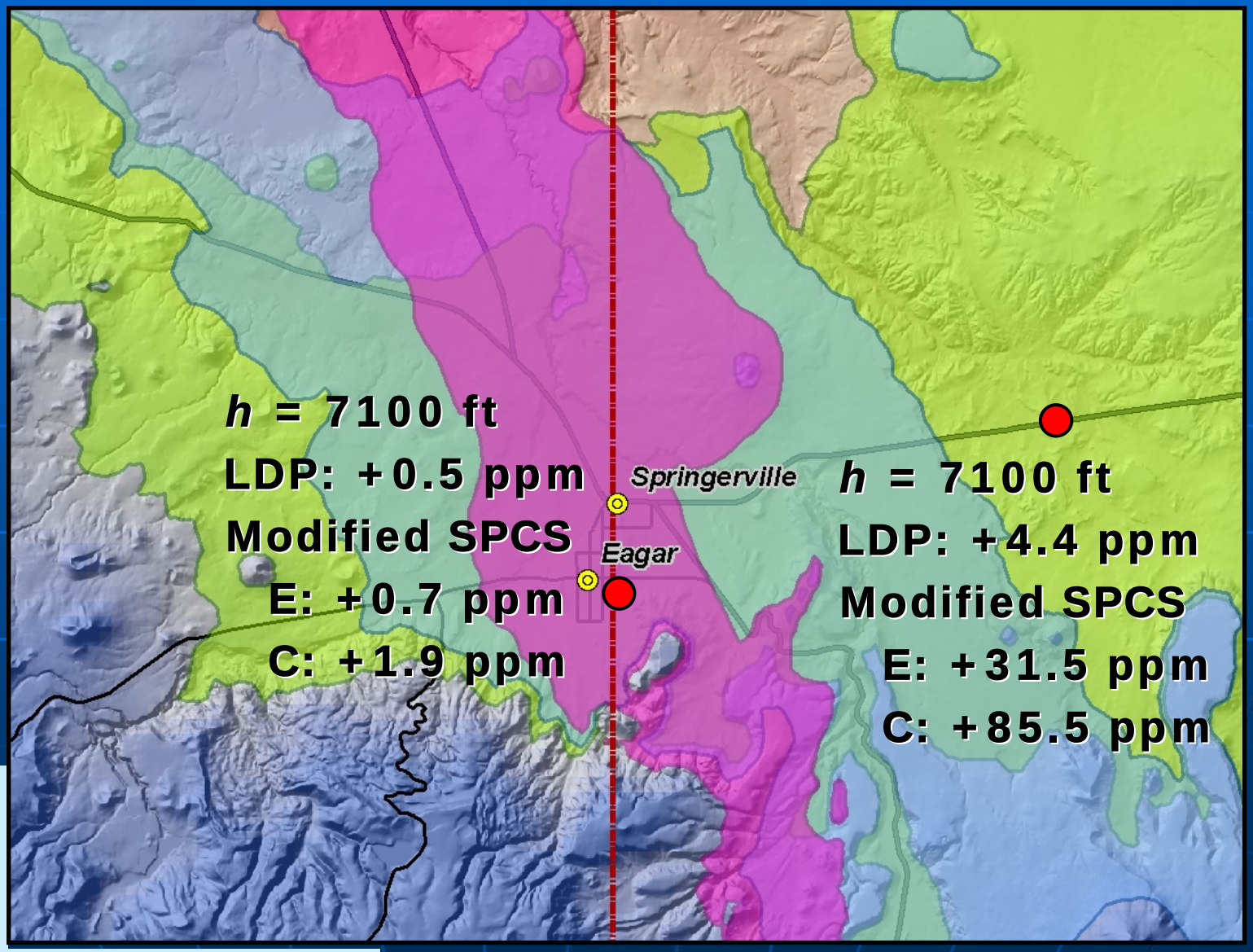
Miles

Linear distortion, ±20 ppm coverage comparison

LDP
±20ppm

Mod SPCS E
±20ppm

Mod SPCS C
±20ppm



60 ppm

-60 ppm

0 2.5 5 10 Miles

Example: Springerville, AZ

Representative ellipsoid height = 7100 ift
Earth radius (geometric mean) = 20,899,500 ift

Projection parameters	Local projection (Transverse Mercator)	"Modified" SPCS 83 AZ E
Origin latitude	34°00'00"N	31°00'00"N
Central meridian longitude	109°17'00"W	110°10'00"W
False northing	0.000 ift	0.000 ift
False easting	50,000.000 ift	700,250.529 473 74... ift
Central meridian scale	1.000 340 (exact)	1.000 257 863 458 28...